On the Outage Performance of Space-Air-Ground Integrated Networks in the 3D Poisson Field

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***Abstract*—Aiming to expand wireless coverage and connect the unconnected, space-air-ground integrated networks (SAGIN) have recently been proposed as a promising paradigm to satisfy the high-rate and high-reliability requirements. SAGIN is comprised by satellite, aerial, and terrestrial communication devices. In this paper, from the perspective of cooperative communications, we regard the SAGIN as a two-hop relay network with the space-air and air-ground cooperative links. Especially, the location distribution of unmanned aerial vehicle (UAV), as known as the communication relay platforms in the SAGIN system, is modeled as a homogeneous Poisson Point Process (PPP) in a three-dimensional (3D) spherical field. We evaluate the outage performance of SAGIN in the 3D Poisson field and approximate its outage probability in closed form. In addition, the parametric study for the approximation technique used to derive the closed-form expression is also provided. Numerical results are presented and discussed to verify our outage performance analysis.**

***Index Terms*—Space-air-ground integrated network, cooperative relaying, three-dimensional Poisson field, outage performance.**

I. Introduction

To realize the 17 Sustainable Development Goals (SDGs) proposed by the United Nations (UN), ubiquitous wireless connectivity with high-rate and high-reliability requirements becomes imperative. Although the conventional terrestrial communications have been widely popularized, there still exists a huge population that is unconnected or under-connected because of the lack of terrestrial infrastructure [1]. Specifically, current networks struggle to offer sufficient broadband coverage to remote areas [2]. Non-terrestrial networks can be crucial complements for providing ubiquitous and just-good-enough connectivity [3]. For instance, unmanned aerial vehicle (UAV) assisted communications may play a key role in extending the communication coverage and providing emergency network recovery [4]. Meanwhile, satellite communication systems can also provide highly reliable data services in planes, trains, ships, and other hard-to-reach regions [5]. By incorporating latest non-terrestrial communication technologies, in recent years space-air-ground integrated network (SAGIN) becomes a promising paradigm to support high-rate and high-reliability data service especially in remote areas [3]. In addition, SAGIN, encompassing the merits of satellite, aerial and terrestrial communications, would greatly facilitate several 6G application scenarios envisioned in [6], such as Enhanced Mobile Broadband Plus (eMBB-Plus), Big Communications (BigCom), and Three-Dimensional Integrated Communications (3D-InteCom).

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In essence, SAGIN is a cooperative relay network, in which UAVs in the aerial layer serves as relay nodes between the space layer and the terrestrial layer and enable a high degree of flexibility for network planning and implementation [2]. As for investigating the performance of SAGIN with a certain specific model, most early works only relied on numerical simulation without deriving an analytical result. Gradually, [7] and [8] started analyzing the performance of SAGIN over free-space optical light-of-sight links. Over time, an increasing amount of analytical research has emerged regarding SAGIN with specific models instead of simple uniform distribution models. For example, [9] developed a cooperative satellite-aerial-terrestrial system model where the location of terrestrial receivers was modeled as a Poisson point process (PPP) in a two-dimensional (2D) plane field. This model enabled the researchers to solve for the outage probability using an approximated expression and optimize the transmit power. Likewise, [10] proposed a cooperative system model consisting of satellites, high-altitude platforms (HAPs), base stations, and user equipment where HAPs can only operate over a certain layer with a fixed height. The authors studied the analytical outage performance first with the PPP study and then worked on the optimization problem by finding the maximum placement distance of the HAP. Moreover, [11] examined a dual-hop cooperative satellite-UAV communication system where uniformly distributed UAVs surround a cluster header, and evaluated its coverage performance. Additionally, [12] investigated a three-dimensional (3D) wireless network comprising several transmitting drones that are uniformly distributed in a finite ball area modeled as a homogeneous binomial point process.

Based on [13] and [14] which researched different areas of UAV characteristics, the PPP model gradually became a widely accepted model to describe UAVs' performance. In this work, we develop a different SAGIN system with the UAVs located in a 3D Poisson field. The main contribution of this paper can be listed as follow.

1) In contrast to previous works such as[9], [10],[11], and [12], we first propose a more realistic SAGIN system model where an indeterminate number of UAVs are randomly distributed over a 3D spherical field and abide by the homogeneous PPP, which enhance the flexibility of communication relay deployment to yield better system reliability.

2) Skillfully using approximation techniques including the Chebyshev-Gauss quadrature method, we obtain the approximate expression of outage probability in closed form for the proposed SAGIN system model. The result of performance analysis can be used in future research for more dynamic conditions as an important benchmark. Meanwhile, by analyzing the approaching derivation process, we also provide the parametric study for determining the proper number of summation terms in the Chebyshev-Gauss quadrature method.

3) We verify the correctness of closed-form expression substantiated by numerical results generated by Monte Carlo simulations. The analytical results and expressions given in this work provide new insights into SAGIN and reveal the impacts of spatial randomness on the reliability of SAGIN.

II. System Model

Let us start from a simplistic two-hop SAGIN model consisting of a geostationary (GEO) satellite, a UAV distributed over a spherical region, and a fixed terrestrial base station (BS). We set the original three-dimensional cartesian coordinates of the GEO satellite, UAV, and terrestrial BS as (x_S, y_S, z_S) , (x_U, y_U, z_U) ,and(x_B, y_B, z_B), respectively. The cartesian coordinates can be transformed to the spherical coordinate regarding the spherical center as the coordinate origin where the transform formula as $x_i = r_i \cos\theta_i \cos\phi_i$, $y_i = r_i \cos\theta_i \sin\phi_i$, and $z_i = r_i \cos\theta_i$ for $i \in \{S, U, B\}$. These result in the form of coordinates as (r_S, θ_S, ϕ_S), (r_U, θ_U, ϕ_U), and (r_B, θ_B, ϕ_B), where the radius of UAVs moving spherical region is R, and we assume $r_S > R$, $r_U < R$, and $r_B > R$, which well fit most practical scenarios. Through the law of cosines, we can easily obtain the distances between satellite and UAV and between UAV and BS, denoted as d_{SU} and d_{UB} in (1) and (2).

In terms of mobility, the GEO satellite, the center of the spherical region, and the BS are stationary, whereas the UAV is mobile, yielding an impact on the reliability of SAGIN. Assuming that the stochastic distribution of the UAV complies with the homogeneous PPP with density λ_0 in a three-dimensional spherical region, θ_U and ϕ_U are circularly and uniformly distributed. The former is from 0 to 2π and the latter is from 0 to π with probability density function (PDF) as $f_{\theta_U}(\lambda) = 1/(2\pi)$ for $0 \le \theta_U < 2\pi$, and $f_{\phi_U}(\lambda) = 1/\pi$ for $0 \le \phi_U < \pi$, respectively.

We denote N as the number of UAVs in sphere V, leading to $P\{N = k\} = \exp(-\zeta_V)(\zeta_V^k/k!)$,where $\zeta_V = \frac{4}{3}\pi R^3 \lambda_0$. As for the distance between coordinate origin and UAVs r_U, it can be calculated from T_i, where T_i represents the ith UAV in the sphere of radius R. As a result, the PDF of T_i can be determined by using the methods developed in [15] as $f_{T_i}(t_i) = \lambda_0/\zeta_V = 3/(4\pi R^3)$, by which we can obtain the cumulative distribution function (CDF) of r_U as

$$
\begin{aligned}
F_{r_U}(x) &= \frac{3}{4\pi R^3} \int_0^x \int_0^\pi \int_0^{2\pi} \sin\phi_u r_U^2 \, \mathrm{d}\theta_U \, \mathrm{d}\phi_U \, \mathrm{d}r_U \\
&= \begin{cases} 0, & x < R \\ x^3/R^3, & 0 \le x \le R \\ 1, & x > R \end{cases}
\end{aligned} \tag{3}
$$

The transmit power of the GEO satellite, UAV, and BS are denoted as P_S, P_U, and P_B, respectively. Meanwhile, we assume that all nodes operate in a half-duplex mode with a single antenna. Without loss of generality, it is also supposed that the GEO satellite, UAV, and BS share the same frequency band (L band or S band). We also assume that UAV adopts the decode-and-forward (DF) relaying protocol when forwarding received messages.

For wireless channel modeling of SAGIN, it has been widely acknowledged that the specific composition of the atmosphere is different in different altitudes, causing different phenomena by various physical mechanisms. These result in diverse communication channel models when modeling SAGIN. Therefore, we are supposed to respectively discuss the different channel models and classify the channels into two categories referring to the channel between GEO satellite and UAV as well as the channel between UAV and BS. For characterizing the channel between GEO satellite and UAV, the Loo channel model is widely recognized and accepted. However, due to the poor mathematical tractability of Loo channel model [16], we adopt the Shadowed Rician fading model [17] to characterize the signal propagation between GEO satellite and UAV. Then, we denote the the channel coefficient between satellite and UAV as h_{SU}. Thus, when $\lambda > 0$, the PDF of the corresponding channel power gain $G_{SU} = |h_{SU}|^2$ can be written as

$$
f_{G_{SU}}(\lambda) = \alpha_{SU} \exp(-\beta_{SU}\lambda) \ {}_1F_1(m_{SU}; 1; \delta_{SU}\lambda), \tag{4}
$$

where $\alpha_{SU} = \left(\frac{2b_{SU}m_{SU}}{2b_{SU}m_{SU}+\Omega_{1SU}}\right)^{m_{SU}} / (2b_{SU})$, $\beta_{SU} = \frac{1}{2b_{SU}}$, and $\delta_{SU} = \frac{\Omega_{SU}}{2b_{SU}(2b_{SU}m_{SU}+\Omega_{SU})}$; Ω_{SU} and b_{SU} are the average power of the light-of-sight (LoS) and multipath components; m_{SU} is the fading severity parameter; ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function of the first kind [18]. As for the channel between UAV and BS, based on the height of the UAV, we regard the signal propagation link between the UAV and the BS as an LoS link and thereby rely on the Rician distribution to describe this channel [19]. Similarly, denoting the channel coefficient between UAV and BS as h_{UB}, the PDF of the channel power gain $G_{UB} = |h_{UB}|^2$ is given by

$$
f_{G_{UB}}(\lambda) = \eta^2 \exp\left(-\left[\eta^2\lambda + K\right]\right) I_0(2\eta\sqrt{K\lambda}), \tag{5}
$$

where $\eta = \sqrt{\frac{1+K}{\Omega_{UB}}}$, K is the Rician factor, Ω_{UB} is the variance of the signal power, and $I_\nu(\cdot)$ is the νth-order modified Bessel function of the first kind.

III. Outage Performance Analysis

A. Definition and Derivation of Exact Outage Probability

Since the whole SAGIN system can be regarded as a two-hop cooperative system, we can simply define the per-link outage probabilities as

$$
\begin{cases} \Phi_{SU}(\epsilon_U) = \mathbb{P}\{P_S G_{SU}/(d_{SU}^{n_{SU}} N_0) < \epsilon_U\} \\ \Phi_{UB}(\epsilon_B) = \mathbb{P}\{P_U G_{UB}/(d_{UB}^{n_{UB}} N_0) < \epsilon_B\} \end{cases}, \tag{6}
$$

where $\mathbb{P}\{\cdot\}$ represents the probability of the random event enclosed; n_{SU} and n_{UB} respectively denotes the path loss exponents for the link between GEO satellite and UAV as well as the link between UAV and BS; ϵ_U and ϵ_B are the outage thresholds at UAV receiver and BS receiver, respectively; N_0 is the average power of the complex additive white Gaussian noise (AWGN). Due to the bottleneck effect, we can thus determine the end-to-end outage probability as $\Phi(\epsilon_U, \epsilon_B) = 1 - (1 - \Phi_{SU}(\epsilon_U))(1 - \Phi_{UB}(\epsilon_B))$.

1) Outage Probability of the GEO Satellite-UAV Link: Based on (4) and (6), we can express the outage probability conditioned on d_{SU} as $\Phi_{SU|d_{SU}}(\epsilon_U) = \mathbb{P}\{G_{SU} < \epsilon_U d_{SU}^{n_{SU}} N_0/P_S\}$ and then derive

$$
\begin{aligned}
\Phi_{SU|d_{SU}}(\epsilon_U) = \int_0^{\epsilon_U d_{SU}^{n_{SU}} N_0/P_S} & \alpha_{SU} \exp(-\beta_{SU}\lambda) \\
& \times {}_1F_1(m_{SU}; 1; \delta_{SU}\lambda) \, \mathrm{d}\lambda.
\end{aligned} \tag{7}
$$

According to the approximate transform formula proven in [20] for hypergeometric function ${}_1F_1(\cdot;\cdot;\cdot)$, we assume m_{SU} takes integer values, which allows expressing the hypergeometirc function part in our formula as

$$d_{SU}\left(r_U,\phi_U,\theta_U\right)=\sqrt{r_S^2+r_U^2-2r_Sr_U\left(\sin(\theta_S)\sin\left(\theta_U\right)\cos\left(\phi_S-\phi_U\right)+\cos(\theta_S)\cos\left(\theta_U\right)\right)} \tag{1}$$

$$d_{UB}\left(r_U,\phi_U,\theta_U\right)=\sqrt{r_B^2+r_U^2-2r_Br_U\left(\sin(\theta_B)\sin\left(\theta_U\right)\cos\left(\phi_B-\phi_U\right)+\cos(\theta_B)\cos\left(\theta_U\right)\right)} \tag{2}$$

$\sum_{k_{SU}=0}^{m_{SU}-1}\frac{(-1)^{k_{SU}}(1-m_{SU})_{k_{SU}}(\delta_{SU}\lambda)^{k_{SU}}}{(k_{SU}!)^2}$. Thus, we can simplify (7) to be

$$\Phi_{SU|d_{SU}}(\epsilon_U)=\int_0^{\epsilon_U d_{SU}^{n_{SU}}N_0/P_S}\alpha_{SU}\exp\left(-\beta_{SU}\lambda\right)\exp\left(\delta_{SU}\lambda\right)\times\sum_{k_{SU}=0}^{m_{SU}-1}\frac{(-1)^{k_{SU}}\left(1-m_{SU}\right)_{k_{SU}}\left(\delta_{SU}\lambda\right)^{k_{SU}}}{\left(k_{SU}!\right)^2}\mathrm{d}\lambda, \tag{8}$$

where $(t)_k=t(t+1)\cdots(t+k-1)$ is the Pochhammer symbol [18]. Since the integral is now given in a simple exponential integral form we can rewrite $\Phi_{SU|d_{SU}}(\epsilon_U)$ in (9), where $\gamma(\cdot,\cdot)$ dontes the incomplete gamma function [18].

Since d_{SU} in (9) is still a random variable, we need to further derive it to a close form independent of d_{SU} by averaging over r_U, θ_U, and ϕ_U. Fortunately, d_{SU} only exists in the incomplete gamma function $\gamma(\cdot,\cdot)$, and we can rewrite the incomplete gamma function $\gamma\left(k_{SU}+1,\left(\beta_{SU}-\delta_{SU}\right)d_{SU}^{n_{SU}}\epsilon_U N_0/P_S\right)$ as $L_{1|d_{SU}(r_U,\phi_U,\theta_U)}$ for simplicity. Due to the mathematical intractability of triple integral of $L_{1|d_{SU}(r_U,\phi_U,\theta_U)}$, the Chebyshev-Gauss quadrature is adopted to facilitate further analysis with an adequate accuracy. Assuming $x_i=\cos\left(\frac{2i-1}{2W}\pi\right)$ and $w_i=\frac{\pi}{W}$, Chebyshev-Gauss quadrature could obtain an adequately fit result of integration by a summation consisting of a finite number of terms as $\int_{-1}^{1}\frac{f(x)}{\sqrt{1-x^2}}dx\approx\sum_{i=1}^{W}w_if\left(x_i\right)$, where W denotes the number of summed terms. Thus, the integral of $L_{1|d_{SU}(r_U,\phi_U,\theta_U)}$ can be approximated to be

$$\int_0^R\int_0^{2\pi}\int_0^{\pi}L_{1|d_{SU}(r_U,\phi_U,\theta_U)}f_{\phi_U}f_{\theta_U}f_{r_U}\mathrm{d}\phi_U \approx\frac{3\pi^3}{8R^2W_1W_2W_3}\sum_{i=1}^{W_1}\sum_{j=1}^{W_2}\sum_{q=1}^{W_3}L_{1|d_{SU}\left(\frac{R}{2}a_i+\frac{R}{2},\pi b_j+\pi,\frac{\pi}{2}c_q+\frac{\pi}{2}\right)}\times\left(\frac{R}{2}a_i+\frac{R}{2}\right)^2\sqrt{(1-a_i^2)(1-b_j^2)(1-c_q^2)}, \tag{10}$$

where $a_i{=}\cos(\frac{2i-1}{2W_1}\pi)$; $b_j{=}\cos(\frac{2j-1}{2W_2}\pi)$; $c_q{=}\cos(\frac{2q-1}{2W_3}\pi)$; W_i denotes the number of summation terms for each transformation from integration to summation by the Gauss Chebyshev method, $i\in 1,2,3,\cdots,9$.

By substituting (10) into (9), we can approximate the outage probability of signal propagation from GEO satellite to UAV in closed form as

$$\Phi_{SU}(\epsilon_U)\approx\alpha_{SU}\sum_{k_{SU}=0}^{m_{SU}-1}\frac{(-1)^{k_{SH}}\left(1-m_{SU}\right)_{k_{SU}}\left(\delta_{SU}\right)^{k_{SU}}}{\left(k_{SU}!\right)^2}\times\left(\beta_{SU}-\delta_{SU}\right)^{-k_{SU}-1}\frac{3\pi^3}{8R^2W_1W_2W_3}\sum_{i=1}^{W_1}\sum_{j=1}^{W_2}\sum_{q=1}^{W_3}\gamma\left(k_{SU}+1,\frac{\left(\beta_{SU}-\delta_{SU}\right)d_{SU^*}^{n_{SU}}\epsilon_UN_0}{P_S}\right)\times\left(\frac{R}{2}a_i+\frac{R}{2}\right)^2\sqrt{(1-a_i^2)(1-b_j^2)(1-c_q^2)}, \tag{11}$$

where $d_{SU^*}=d_{SU}\left(\frac{R}{2}a_i+\frac{R}{2},\pi b_j+\pi,\frac{\pi}{2}c_q+\frac{\pi}{2}\right)$.

2) Outage Probability of the UAV-BS Link: According to (5), the outage probability conditioned on d_{UB} can be rewritten as $\Phi_{UB|d_{UB}}(\epsilon_B)=\mathbb{P}\left\{G_{UB}<\epsilon_Hd_{UB}^{n_{UB}}N_0/P_U\right\}$ and computed in closed form as

$$\Phi_{UB|d_{UB}}(\epsilon_B)=1-Q_1\left(\sqrt{2K},\eta\sqrt{\frac{2\epsilon_BN_0d_{UB}^{n_{UB}}}{P_U}}\right). \tag{12}$$

Considering the distribution of d_{UB} in terms of r_U, ϕ_U, and θ_U, we can approximate the outage probability of the UAV-BS link by using Chebyshev-Gauss quadrature method three times, which is similar as the process in (10), yielding the following closed-form expression:

$$\begin{aligned}\Phi_{UB}(\epsilon_B)\approx&1-\int_0^R\int_0^{2\pi}\int_0^{\pi}Q_1\left(\sqrt{2K},\eta\sqrt{\frac{2\epsilon_BN_0d_{UB}^{n_{UB}}}{P_U}}\right)\times 3\pi/\left(4\pi^2R^3\right)\mathrm{d}\phi_U\mathrm{d}\theta_U\mathrm{d}r_U\\=&1-\frac{3\pi}{8R^3W_4W_5W_6}\sum_{i=1}^{W_4}\sum_{j=1}^{W_5}\sum_{q=1}^{W_6}Q_1\left(\sqrt{2K},\eta\sqrt{\frac{2\epsilon_BN_0d_{UB^*}^{n_{UB}}}{P_U}}\right)\times\left(\frac{R}{2}x_i+\frac{R}{2}\right)^2\sqrt{(1-x_i^2)(1-y_j^2)(1-z_q^2)},\end{aligned} \tag{13}$$

where $d_{UB^*}=d_{UB}\left(\frac{R}{2}x_i+\frac{R}{2},\pi y_j+\pi,\frac{\pi}{2}z_q+\frac{\pi}{2}\right)$; $x_i=\cos(\frac{2i-1}{2W_4}\pi)$; $y_j=\cos(\frac{2j-1}{2W_5}\pi)$; $z_q=\cos(\frac{2q-1}{2W_6}\pi)$.

Furthermore, we can simplify above calculation by approximating Q_1 function as $Q_1(x,y)\approx\exp\left(-\exp\left(\omega(x)\right)y^{\tau(x)}\right)$, where $\omega(x)=-0.840+0.327x-0.740x^2+0.083x^3-0.004x^4$ and $\tau(x)=2.174-0.592x+0.593x^2-0.092x^3+0.005x^4$ [21]. As a result, the approximated outage probability of signal propagation from UAV to BS can be written as

$$\Phi_{UB}(\epsilon_B)\approx1-\frac{3\pi^3}{8R^3W_4W_5W_6}\sum_{i=1}^{W_4}\sum_{j=1}^{W_5}\sum_{q=1}^{W_6}\exp\left(-\exp\left(\omega(\sqrt{2K})\right)\eta\sqrt{\frac{2\epsilon_BN_0d_{UB^*}^{n_{UB}}}{P_U}}^{\tau(\sqrt{2K})}\right)\times\left(\frac{R}{2}x_i+\frac{R}{2}\right)^2\sqrt{(1-x_i^2)(1-y_j^2)(1-z_q^2)}. \tag{14}$$

3) End-to-end Outage Probability: According to $\Phi\left(\epsilon_U,\epsilon_B\right)=1-(1-\Phi_{SU}(\epsilon_U))(1-\Phi_{UB}(\epsilon_B))$, we can derive the end-to-end outage probability through (11) and (14) and obtain its closed-form approximation in (15), where $u_i=cos(\frac{2i-1}{2W_7}\pi)$; $v_j=cos(\frac{2j-1}{2W_8}\pi)$; $t_q=\cos(\frac{2q-1}{2W_9}\pi)$; $d_{SU^{**}}=d_{SU}\left(\frac{R}{2}u_i+\frac{R}{2},\pi v_j+\pi,\frac{\pi}{2}t_q+\frac{\pi}{2}\right)$; $d_{UB^{**}}=d_{UB}\left(\frac{R}{2}u_i+\frac{R}{2},\pi v_j+\pi,\frac{\pi}{2}t_q+\frac{\pi}{2}\right)$.

B. Parametric Study

Based on the calculations presented in the previous subsection, we predominantly relied on Chebyshev-Gauss quadrature

$$\Phi_{SU|d_{SU}}(\epsilon_U) = \alpha_{SU} \sum_{k_{SU}=0}^{m_{SU}-1} \frac{(-1)^{k_{SU}} (1-m_{SU})_{k_{SU}} (\delta_{SU})^{k_{SU}}}{(k_{SU}!)^2} (\beta_{SU}-\delta_{SU})^{-k_{SU}-1} \gamma\left(k_{SU}+1, \frac{(\beta_{SU}-\delta_{SU}) d_{SU}^{n_{SU}} \epsilon_U N_0}{P_S}\right) \tag{9}$$

$$\Phi(\epsilon_U,\epsilon_B) \approx 1 - \frac{3\pi^3}{8R^2 W_7 W_8 W_9} \sum_{i=1}^{W_7}\sum_{j=1}^{W_8}\sum_{q=1}^{W_9}\sum_{k_{SU}=0}^{m_{SU}-1} (\frac{R}{2}u_j + \frac{R}{2})^2 \sqrt{(1-u_j^2)(1-v_q^2)(1-t_i^2)} \exp\left(-\exp\left(\omega(\sqrt{2K})\right)\eta\sqrt{\frac{2\epsilon_B N_0 d_{UB^{**}}^{n_{UB}}}{P_U}}^{\tau(\sqrt{2K})}\right)$$
$$\times\left(1-\alpha_{SU}\frac{(-1)^{k_{SH}}(1-m_{SU})_{k_{SU}}(\delta_{SU})^{k_{SU}}}{(k_{SU}!)^2}(\beta_{SU}-\delta_{SU})^{-k_{SU}-1}\gamma\left(k_{SU}+1,\frac{(\beta_{SU}-\delta_{SU}) d_{SU^{**}}^{n_{SU}}\epsilon_U N_0}{P_S}\right)\right) \tag{15}$$

to approximate the integration operation in (10) and (13). Essentially, the Chebyshev-Gauss quadrature approximation can be regarded as dividing the area of integration into multiple rectangle areas for summation. Normally, with the increase of the summation terms, the approximate result will gradually approach the exact calculation. To better manage the trade-off between accuracy and computational complexity, we need to find a method to obtain proper numbers of summation terms.

From [22], the error of the first-kind Gauss–Chebyshev quadrature rules, denoted as $E(f)$, can be quantified through the following relation:

$$\begin{aligned}\int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}}\,\mathrm{d}x = \frac{\pi}{W}\sum_{k=1}^{W} f\left(\cos\left(\frac{(2k-1)\pi}{2W}\right)\right) \\ + \underbrace{\frac{2\pi}{2^{2W}(2W)!} f^{(2W)}(\mu)}_{E(f)}, \quad -1<\mu<1,\end{aligned} \tag{16}$$

where $f^{(n)}(x)$ denotes the n-th derivative of a function $f(x)$. Referring back to (10) and normalizing all constant coefficients, we obtain $\Xi_1(x) = \gamma(k_{SU}+1, x^2+x)\,(x+1)^2\sqrt{1-x^2}$, where we also assume $n_{SU}=2$ as an example. Based on the general Leibniz rule, we further obtain the following expression:

$$\Xi_1{}^{(2W_1)}(x) = \sum_{k=0}^{2W_1}\binom{2W_1}{k} g_1^{(2W_1-k)}(x) l_1^{(k)}(x), \tag{17}$$

where $g_1(x) = \gamma(k_{SU}+1, x^2+x)\sqrt{1-x^2}$ and $l_1(x) = (x+1)^2$. Denoting $p_1(x) = \gamma(k_{SU}+1, x^2+x)$ and $l_2(x) = \sqrt{1-x^2}$, we can express the derivation form in terms of Bell polynomials $B_{n,i}(x_1,\ldots,x_{n-i+1})$ as

$$p_1^{(n)}(x) = \sum_{i=1}^{n} \frac{\mathrm{d}^i\gamma(k_{SU}+1, x^2+x)}{\mathrm{d}(x^2+x)^i} B_{n,i}\left(2x, 2, \underbrace{0\ldots,0}_{n-i-1}\right), \tag{18}$$

which leads to

$$g_1{}^{(2W_1)}(x) = \sum_{k=0}^{2W_1}\binom{2W_1}{k} p_1^{(2W_1-k)}(x) l_2^{(k)}(x), \tag{19}$$

When $W>2$, $l_1^{(W)}=0$, resulting in

$$\begin{aligned}\Xi_1{}^{(2W_1)}(x) = g_1^{(2W_1)}(x) l_1^{(0)}(x) + 2W_1 g_1^{(2W_1-1)}(x) l_1^{(1)}(x) + \\ 2W_1(2W_1-1) g_1^{(2W_1-2)}(x) l_1^{(2)}(x).\end{aligned} \tag{20}$$

Then, we can set a threshold guaranteeing $\Psi(W_1) > E(\Xi_1) = \frac{2\pi}{2^{2W_1}(2W_1)!}\Xi^{(2W_1)}(\mu)$ for controlling the error bound for each summation. By selecting a proper $\Psi(W_1)$ corresponding to the accuracy requirement, we can obtain an appropriate value of W_1.

Similarly, we denote $g_2(x) = \gamma(k_{SU}+1, \cos x)$ and $g_3(x) = \gamma(k_{SU}+1, \cos x + \sin x)$, leading to $\Xi_2(x) = \gamma(k_{SU}+1, \cos x)\sqrt{1-x^2}$ and $\Xi_3(x) = \gamma(k_{SU}+1, \cos x + \sin x)\sqrt{1-x^2}$, which can be approximated as follows:

$$\Xi_2{}^{(2W_2)}(x) = \sum_{k=0}^{2W_2}\binom{2W_2}{k} g_2^{(2W_2-k)}(x) l_2^{(k)}(x), \tag{21}$$

$$\Xi_3{}^{(2W_3)}(x) = \sum_{k=0}^{2W_3}\binom{2W_3}{k} g_3^{(2W_3-k)}(x) l_2^{(k)}(x), \tag{22}$$

where

$$\begin{aligned}g_2^{(n)}(x) = \sum_{i=1}^{n} \frac{\mathrm{d}^i\gamma(k_{SU}+1, \cos x)}{\mathrm{d}(\cos x)^i} \\ \times B_{n,i}\left(\underbrace{-\sin x, -\cos x, \sin x, \cos x, -\sin x \ldots}_{n-i+1}\right),\end{aligned} \tag{23}$$

and $g_3^{(n)}(x)$ is shown in (24). Following the above calculations, $E(\Xi_2)$ and $E(\Xi_3)$ can also be used to obtain the value of W_2 and W_3 apropos of thresholds $\Psi(W_2)$ and $\Psi(W_3)$.

As for the link from UAV to BS, we similarly assume that $n_{UB}=2$ and $\tau(\sqrt{2K})=2$ and determine the following relations: $\Xi_4(x) = \exp(x^2+x)\,(x+1)^2\sqrt{1-x^2}$, $\Xi_5(x) = \exp(\cos x)\sqrt{1-x^2}$, and $\Xi_6(x) = \exp(\cos x + \sin x)\sqrt{1-x^2}$. Denoting $g_4(x) = \exp(x^2+x)\sqrt{1-x^2}$, $p_4(x) = \exp(x^2+x)$, $g_5(x) = \exp(\cos x)$, and $g_6(x) = \exp(\cos x + \sin x)$, we then follow the same steps to obtain

$$\begin{aligned}\Xi_4{}^{(2W_4)}(x) = g_4^{(2W_4)}(x) l_1^{(0)}(x) + 2W_1 g_4^{(2W_4-1)}(x) l_1^{(1)}(x) \\ + 2W_4(2W_4-1) g_4^{(2W_4-2)}(x) l_1^{(2)}(x),\end{aligned} \tag{25}$$

$$\Xi_5{}^{(2W_5)}(x) = \sum_{k=0}^{2W_5}\binom{2W_5}{k} g_5^{(2W_5-k)}(x) l_2^{(k)}(x), \tag{26}$$

$$\Xi_6{}^{(2W_6)}(x) = \sum_{k=0}^{2W_6}\binom{2W_6}{k} g_6^{(2W_6-k)}(x) l_2^{(k)}(x), \tag{27}$$

where

$$p_4^{(n)}(x) = \sum_{i=1}^{n} \exp(x^2+x) B_{n,i}\left(2x, 2, \underbrace{0\ldots,0}_{n-i-1}\right), \tag{28}$$

$$g_4{}^{(2W_4)}(x) = \sum_{k=0}^{2W_4}\binom{2W_4}{k} p_4^{(2W_4-k)}(x) l_2^{(k)}(x); \tag{29}$$

$g_5^{(n)}(x)$ and $g_6^{(n)}(x)$ are shown in (30) and (31). Thus, $W_i, i \in \{4, 5, \ldots, 9\}$ can all be obtained corresponding to specific threshold $\Psi(W_i)$.

$$g_3^{(n)}(x) = \sum_{i=1}^{n} \frac{\mathrm{d}^i \gamma(k_{SU}+1, \cos x + \sin x)}{\mathrm{d}(\cos x + \sin x)^i} B_{n,i}\left(\underbrace{\cos x - \sin x, -\cos x - \sin x - \cos x + \sin x, \cos x + \sin x, \cos x - \sin x \ldots}_{n-i+1}\right) \tag{24}$$

$$g_5^{(n)}(x) = \sum_{i=1}^{n} \exp\left(\cos x\right) B_{n,i}\left(\underbrace{-\sin x, -\cos x, \sin x, \cos x, -\sin x \ldots}_{n-i+1}\right) \tag{30}$$

$$g_6^{(n)}(x) = \sum_{i=1}^{n} \exp\left(\cos x + \sin x\right) B_{n,i}\left(\underbrace{\cos x - \sin x, -\cos x - \sin x - \cos x + \sin x, \cos x + \sin x, \cos x - \sin x \ldots}_{n-i+1}\right) \tag{31}$$

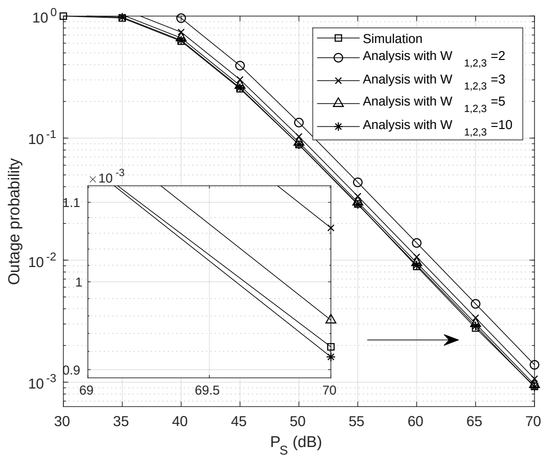

Fig. 1: Outage probability of the link between GEO satellite and UAVs versus the transmit power of GEO satellite.

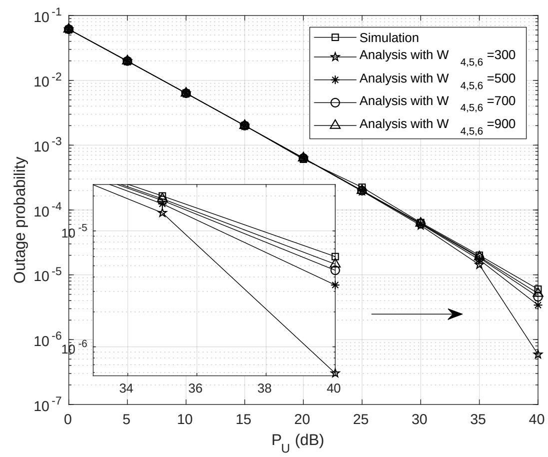

Fig. 2: Outage probability of the link between UAVs and terrestrial BS versus the transmit power of UAVs.

IV. Simulation Results and Discussion

To explore the reliability of SAGIN and verify the analysis presented in the last section, Monte Carlo simulations were used to produce numerical results, which are shown and discussed in this section. Based on the parameters provided from [23], the main adopted parameters in our model are set as $\epsilon_U = \epsilon_B = 1$dB, $n_{SU} = n_{UB} = 2$, $m_{SU} = m_{UB} = 2$, $b_{SU} = b_{UB} = \Omega_{SU} = \Omega_{UB} = 15$ dB, $K = -10$ dB, and $N_0 = -94$ dB. As for the locations of the GEO satellite, the center of the UAV distribution sphere, and the BS, we take real-world application scenarios into account and set them as $(x_S, y_S, z_S) = (2,373,639,35,786,000), (x_U, y_U, z_U) = (2,443,189,20,000)$, and$(x_B, y_B, z_B) = (34,199,0)$ with radius $R = 2,000$[1].

Fig. 1 shows the outage performance of the link between the GEO satellite and UAVs. When the power of satellite P_S is less than 30 dB, effective data communications is hardly sustainable. The outage probability gradually reduces from 1 to 10^{-3} when the power of satellite P_S changes from 30 dB to 70 dB. Although the channel quality is still not satisfactory to maintain the communication process when P_S is less than 40 dB, the curve sharply goes down when P_S >40 dB and finally achieves 10^{-3}. Fig. 1 also displays and validates the variety of calculation errors by using Chebyshev-Gauss quadrature with different numbers of summation terms. As expected, the analytical curves almost overlap with the numerical curves when $W_{1,2,3} = 10$, which is computationally efficient for most practical applications. The impact of P_U, the transmit power of UAVs, on the reliability of SAGIN is presented in Fig. 2. Different from the first hop, the link from UAVs to the BS demonstrates an almost linear decreasing curve with the increase of P_U. As shown in this figure, the second hop achieves a low outage probability down to 10^{-5} when P_U =40 dB. However, the computational complexity caused by using the Chebyshev-Gauss Quadrature method is rather high for achieving an adequate accuracy. Even having split the entire integration area into 500 subareas, the difference between the analytical results and the numerical results is still obvious when P_U >35 dB.

Fig. 3 provides a similar set of curves as Fig. 1. The numbers of summation terms $W_{7,8,9}$ only need to be small values for achieving lower outage probabilities. Since the distance from the satellite to UAVs is much greater than that from the UAVs to the ground BS, the outage events always occur over the first hop. That is, the first hop between the GEO satellite and UAVs dominates the outage performance of SAGIN.

Fig. 4 shows the simulation time with different iteration numbers via different links. All data were generated on a workstation as Intel® Xeon(R) W-2145 CPU@3.70GHz ×16 using 31.3 GiB of RAM. The large summation iteration comes with a high calculation time, especially for the link between

[1]GEO satellite is at an altitude of 35,786 km, while the altitude of solar-power UAVs can be over 20,000m [24].

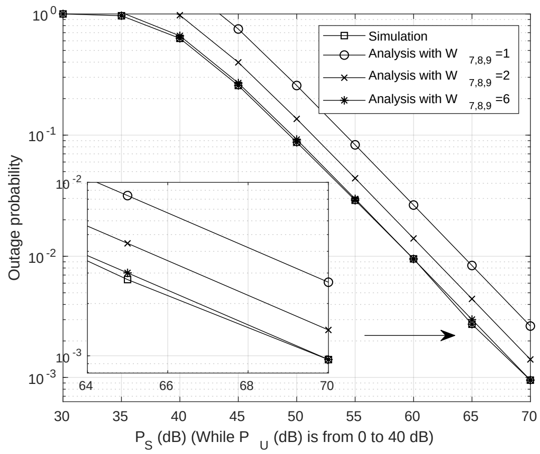

Fig. 3: End-to-end outage probability versus the transmit power of GEO satellite.

Iteration number	2	3	5
Link between GEO satellite and UAVs	0.2718s	2.3198s	10.1786s
Iteration number	300	500	700
Link between UAVs and terrestrial BS	60.6142s	5.178×10^2s	1.6244×10^3s
Iteration number	1	2	6
End-to-end link	0.9854s	4.9010s	15.1108s.

Fig. 4: Calculation time(seconds) for different link

UAVs and terrestrial BS. The trade-off between calculation time with result accuracy will always be a challenge.

Overall, the simulation results indicate that as the number of summation terms increases, the proposed approximation becomes accurate. With a sufficiently large number of summation iterations, the analytical curve can closely match the simulation curve, thereby validating its correctness. This analytical closed-form solution could serve as a valuable tool for future research on similar models. The optimal number of terms for different parameter configurations may vary, posing a challenge for optimizing algorithms under normalized setups.

V. Conclusion

In this paper, we modeled SAGIN as a two-hop relay network, within which the location distribution of UAVs is a homogeneous PPP over a 3D spherical region. Following this model and the basic assumptions associated, we investigated the outage performance of SAGIN and approximate the outage probability in closed form with the help of the Chebyshev-Gauss quadrature method. In addition, we provided the parametric study for determining the numbers of summation terms when using Chebyshev-Gauss quadrature for approximating three-fold integration. All analytical results presented in this paper have been well supported by the numerical results generated by Monte Carlo simulations.

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