

EMF-Aware Probabilistic Shaping Design for Hardware Distorted Communication Systems

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2 ABSTRACT

3 The fifth-generation cellular network requires dense installation of radio base stations (BS) to
4 support ever-increasing demands of high throughput and coverage. The ongoing deployment
5 has triggered some health concerns among the community. To address this uncertainty, we
6 propose an EMF-aware probabilistic shaping design for hardware distorted communication
7 systems. The proposed scheme aims to minimize human exposure to radio-frequency (RF)
8 radiations while achieving the target throughput using probabilistic shaping. The joint optimization
9 of the transmit power and non-uniform symbol probabilities is a non-convex optimization problem.
10 Therefore, we employ alternate optimization and successive convex approximation to solve the
11 subsequent problems. Our findings reveal a significant reduction in the users' exposure to EMF
12 while achieving the requisite quality-of-service with the help of probabilistic shaping in a hardware
13 distorted communication system.

14 **Keywords:** Asymmetric signaling, Error probability analysis, Hardware impairments, Improper noise, Non-uniform probabilities,
15 Radiation exposure

1 INTRODUCTION

16 The next-generation wireless communication networks are expected to fulfill the ever-increasing demands
17 of higher data rates, ultra-reliability, minimal latency, high energy efficiency, and massive connectivity for
18 many users/devices (Latva-aho et al. (2020)). The fifth-generation (5G) is envisioned to support numerous
19 diverse services, such as enhanced mobile broadband (eMBB), ultra-reliable low latency communication
20 (URLLC) and massive machine-type communication (mMTC) (Wan et al. (2018)). Some of the new
21 spectrum allocated for 5G deployments, e.g., millimeter waves, suffers from relatively high path loss
22 limiting the coverage area. Thus, network densification becomes essential to achieve the promised data
23 rate, which can be realized over space (e.g., dense deployment of base station (BS)s in small cells) and
24 frequency (large segments of RF spectrum in diverse bands) (Bhushan et al. (2014)).

25 Installation of the 5G cellular technology with extreme node and network densification (with BSs being
26 closer to users) is raising health concerns about the impact of electric and magnetic fields (EMF) exposure
27 on the population. These worries have sparked several protests against 5G technology and led to some

28 attacks on the 5G BSs (Elzanaty et al. (2021)). Recently, anti-5G protests were held in 30 countries around
29 the world against the threat of 5G wireless technology to public health, the environment, and privacy.

30 Indeed, the thermal effect is the only proven health impact from the RF non-ionized short-term
31 exposure. Therefore, it is necessary to keep the radiation intensity below specific values defined by the
32 exposure regulations and guidelines, e.g., Federal Communications Commission (FCC) and International
33 Commission on Non-Ionizing Radiation Protection (ICNIRP) (Buchner and Rivasi (2020)). Nevertheless,
34 there is a debate about the severe health impacts due to long-term exposure to EMF (National Toxicology
35 Program (2018b,a); Vornoli et al. (2019)). Hence, the International Agency for Research on Cancer (IARC)
36 classified RF radiation as “possibly carcinogenic to humans” (Vornoli et al. (2019); Group (2013); Wilbourn
37 et al. (1986)).

38 Recently, a comprehensive study revealed that the exposure due to the uplink (UL) from the user
39 equipment (UE) is higher than that from the BS due to the proximity of the UE to the user (Lou et al.
40 (2021)). Another work has proposed an architectural solution to minimize the EMF exposure using
41 Reconfigurable intelligent surfaces (Ibraiwish et al. (2022)). Further study is focused on the meticulous
42 cellular network planning to limit the exposure from BSs exploiting MIMO while ensuring coverage and
43 capacity constraints (Matalatala et al. (2018)). From regulatory aspects, some possible risk mitigation
44 strategies are the dismissal of legacy 2G/3G/4G technologies and reduction of emission from non-cellular
45 sources (Chiaraviglio et al. (2021)).

46 Nevertheless, as mentioned earlier, the EMF exposure-based research does not consider any hardware
47 impairments, which can drastically affect the system performance, such as raising the noise floor. For
48 instance, in order to reach the target data rates in eMBB, the hardware distortion (HWD) should be
49 mitigated. Generally, the user will send much higher power, escalating their RF exposure, to achieve the
50 target rate. The users may still not reach the required throughput due to the saturation for the data rate at
51 higher SNR as the distortion noise increases with the transmit power.

52 HWD requires some meticulously designed systems to compensate for its effect and mitigate the
53 performance loss. improper Gaussian signaling (IGS) is an effective compensation signaling scheme
54 that can alleviate the impact of several interference and imperfection sources (Javed et al. (2019)). However,
55 realizing the IGS comes with the inherent problems of unbounded peak-to-average power ratio and high
56 detection complexity (Santamaria et al. (2018); Javed et al. (2020)). As a consequence, researchers
57 employ finite discrete asymmetric signaling (AS) schemes that can be achieved by geometric shaping (GS),
58 probabilistic shaping (PS), or hybrid shaping (HS) (Elzanaty and Alouini (2022); Javed et al. (2021)). In
59 this work, we propose an asymmetric signaling by adopting PS to tackle improper HWD and minimize
60 EMF exposure to the users while maintaining throughput quality of service (QoS). The contributions of
61 this paper are summarized as follows:

- 62 • We model HWD and EMF exposure in the next generation wireless cellular network. We present
63 appropriate receiver and rigorous error probability analysis considering the improper distortion noise.
- 64 • We propose probability shaping as a form of asymmetric signaling to effectively mitigate improper
65 HWDs and reduce EMF exposure while maintaining QoS in terms of user throughput.
- 66 • We employ alternate optimization to jointly design the transmit power of users and non-uniform
67 symbol probabilities. We further use the successive convex approximation and the Newton-Raphson
68 method to solve the subsequent problems.

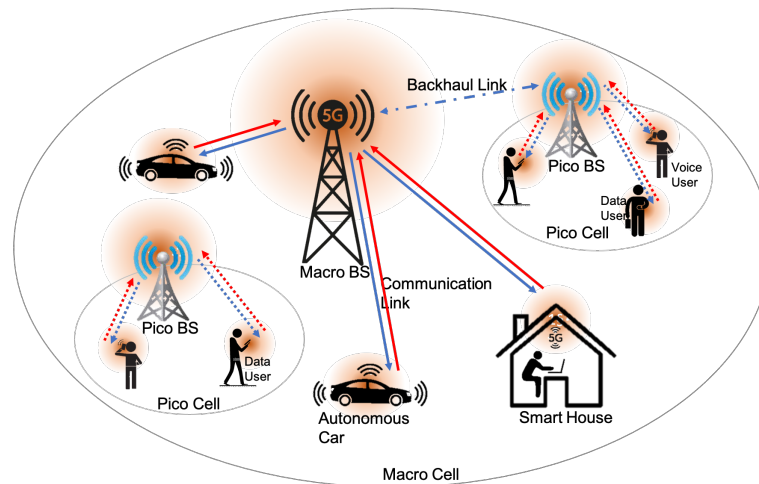


Figure 1. EMF Exposure from 5G Network Infrastructure

69 The rest of the paper is organized such that Section 2 illustrates the system description and adopted
 70 models to characterize the radiation exposure and HWD. Error probability analysis is carried out in Section
 71 3, whereas Section 4 covers the problem formulation and optimization framework. Numerical results are
 72 presented in Section 5, followed by the conclusion in Section 6.

2 SYSTEM DESCRIPTION

73 In this section, we quantify the EMF-exposure of the human population in a next-generation wireless
 74 cellular communication system. The network comprises of macro-cell with a macro BS and multiple pico-
 75 cells with their serving pico BSs. Apart from the transmitting radiations from the BSs in downlink (DL), we
 76 are interested in the EMF exposure caused by the handheld devices. For instance, the RF radiations emitting
 77 from the cell phones and smart watches are stronger near the users resulting in the near-field exposure,
 78 whereas the EMF transmission density from BS towers resides far from the users rendering far-field
 79 exposure. Likewise, increasing applications and use cases including but not limited to autonomous vehicles
 80 and smart homes/offices, etc., are extending the radiation footprint near humans. Figure 1 demonstrates
 81 a strong radiation pattern near users in the case of voice users compared to data users. In addition, we
 82 account for the performance degradation caused by the additive HWDs accumulating from various blocks in
 83 non-ideal RF transceivers. We propose a generalized digital communication system capable of transmitting
 84 non-uniformly distributed symbols (from a uniform bitstream using distribution matching) while employing
 85 an appropriate receiver for optimal detection.

86 2.1 Radiation Exposure

87 The modeling and quantification of human exposure in a wireless cellular network are critical to facilitate
 88 the reduction efforts. Numerous studies have identified the essential parameters influencing this end
 89 exposure (Kuehn et al. (2019); Lou et al. (2021); Vermeeren et al. (2015a)). The EMF exposure, quantified
 90 as Exposure Index (EI), is primarily dictated by the network topology, environment, radio access technology,
 91 user scenarios, and service types, etc. (Kuehn et al. (2019)). The EI can be written as the weighted sum of

92 all branches in the chain of exposure (Lou et al. (2021))

$$93 \quad \text{EI} = \sum_g \sum_p \sum_e \sum_r \sum_l \sum_s f \left(\text{SAR}^{\text{UL}}, \alpha, \text{SAR}^{\text{DL}}, \rho \right) \quad (1)$$

94 where specific absorption rate (SAR), measured in W/Kg, is the power absorbed per mass of the exposed
 95 tissue for a given period with SAR^{UL} and SAR^{DL} indicating the normalized values of the UL and DL
 96 induced SAR when the mean transmit power $\alpha = 1\text{W}$ and the mean received power density $\rho = 1\text{W}/\text{m}^2$,
 97 respectively. The EI is integrated over different age groups g (e.g., children, young people, adults, and
 98 seniors), user posture p (e.g., standing, sitting), environments e (e.g., indoor, outdoor, commuting), radio
 99 access technology r (e.g., GSM, UMTS, WiFi, 5G), layers l (macro, micro, pico, femto), service/usage type
 100 s (e.g., voice and data) (Tesanovic et al. (2014)). We consider an exposure scenario of the next-generation
 101 cellular network in an indoor environment where the reference SAR is averaged over different population
 102 ages and postures, accommodating both data and voice usage types. The EI for both UL and DL can be
 103 given by $\text{EI} = \text{EI}^{\text{UL}} + \text{EI}^{\text{DL}}$, combining the DL exposure induced all day long by base stations/access
 104 points and the UL exposure incurred by individual wireless communication devices.

104 Contrary to the general perception, the SAR from UL renders the dominant part instead of the one
 105 from DL with low ρ (generally $\leq 10\text{mW}/\text{m}^2$ according to FCC) given a significant distance between the
 106 transmitter and receiver. Thus, the exposure index (EI) metric in the presented scenario is dominated by the
 107 UL as $\text{EI}(\alpha) = \text{SAR}^{\text{UL}}\alpha$, the reference whole body or localized SAR mainly depends on the required
 108 service and the posture (Vermeeren et al. (2015b); Lou et al. (2021)).

109 2.2 Hardware Impaired Signal Model

110 Non-linear transfer functions of various transmitter RF stages, such as digital-to-analog converter, band-
 111 pass filter, and high power amplifier, resulting in additive distortion noise η_t , which is distributed as a
 112 zero-mean complex Gaussian random variable $\eta_t \sim \mathcal{CN}(0, \kappa_t, \tilde{\kappa}_t)$ with variance κ_t and pseudo-variance $\tilde{\kappa}_t$.
 113 The complete statistical characterization requires pseudo-variance, in addition to the variance, accounting
 114 for the correlated and/or unequal power distribution among quadrature components of the general complex
 115 Gaussian random variable (Javed et al. (2017)). Importantly, the value of pseudo-variance is limited by the
 116 variance as $|\tilde{\kappa}_t| \leq \kappa_t$ (Björnson et al. (2013); Schenk (2008)).

The Gaussian model for the aggregate residual RF distortions is based on various theoretical investigations,
 including the central limit theorem and measurement results after applying existing compensation schemes
 (Wenk (2010); Zetterberg (2011); Boulogeorgos et al. (2016); Xia et al. (2015); Suzuki et al. (2008);
 Studer et al. (2010); Duy et al. (2015); Björnson et al. (2014) and references therein). This can also be
 motivated analytically by the central limit theorem. The accumulative distortions raise the noise floor of the
 transmitted signal as $x_{\text{tx}} = x_m + \eta_t$, where x_m is the single-carrier band-pass modulated signal taken from
 M -ary QAM, M -ary PSK, or M -ary PAM constellation with a probability mass function $p_m \triangleq p_X(x_m)$,
 rendering the transmission probability of symbol x_m , and $\mathbf{p} \triangleq [p_1, p_2, \dots, p_M]$. Let us define the set that
 includes all possible symbol distributions as

$$\mathbb{S} = \left\{ \mathbf{p} : \mathbf{p} = [p_1, p_2, \dots, p_M], \sum_{j=1}^M p_j = 1, p_j \geq 0, \forall j \in \{1, 2, \dots, M\} \right\}.$$

117 The information-bearing signal is transmitted with average power α and received under additive white
 118 Gaussian noise (AWGN) condition and receiver distortions $\eta_r \sim \mathcal{CN}(0, \alpha \kappa_r, \alpha \tilde{\kappa}_r)$. These distortions

119 result from the non-linear transfer function of low noise amplifier, band-pass filters, image rejection low
 120 pass filter, and analog-to-digital converter at the receiver. Thus, the received signal in point to point (P2P)
 121 communication under improper HWD can be modeled as

$$y = \sqrt{\alpha}x_m + \sqrt{\alpha}\eta + w; \quad m = 1, 2, \dots, M \quad (2)$$

122 where $w \sim \mathcal{CN}(0, \sigma_w^2, 0)$ is the thermal noise and the aggregate effect of transceiver distortions is
 123 represented by, $\eta \sim \mathcal{CN}(0, \kappa, \tilde{\kappa})$, $\kappa = \kappa_t + \kappa_r$ and $\tilde{\kappa} = \tilde{\kappa}_t + \tilde{\kappa}_r$. Interestingly, the generalized impropriety
 124 characterization assists in accurate system modeling, rigorous performance analysis, and appropriate
 125 signaling design. Interested readers can study (Javed et al. (2020)) for the details of statistical impropriety
 126 characterization.

127 2.3 Noise Distribution and Optimal Receiver

Considering the aggregate noise $z = \sqrt{\alpha}\eta + w$, distributed as $z \sim \mathcal{CN}(0, \alpha\kappa + \sigma_w^2, \alpha\tilde{\kappa})$ where in-phase
 z_I and quadrature-phase z_Q noise components are distributed with the respective variances as

$$\sigma_I^2 = (\alpha(\kappa + \Re(\tilde{\kappa})) + \sigma_w^2) / 2, \quad (3)$$

$$\sigma_Q^2 = (\alpha(\kappa - \Re(\tilde{\kappa})) + \sigma_w^2) / 2, \quad (4)$$

128 These individual variances are obtained by simultaneously solving the equations of variance, i.e.,
 129 $\mathbb{E}\{|z|^2\} = \sigma_I^2 + \sigma_Q^2$ and pseudo-variance $\mathbb{E}\{z^2\} = \sigma_I^2 - \sigma_Q^2 + 2ir_{z_I z_Q}$. Thus, the correlation
 130 $r_{z_I z_Q} = \alpha\Im(\tilde{\kappa}) / 2$, defines the correlation coefficient ρ_z between z_I and z_Q as

$$\rho_z = \frac{r_{z_I z_Q}}{\sigma_I \sigma_Q} = \frac{\alpha\Im(\tilde{\kappa})}{\sqrt{(\alpha\kappa + \sigma_w^2)^2 - (\alpha\Re(\tilde{\kappa}))^2}}. \quad (5)$$

131 Unequal power distribution among quadrature noise components and non-trivial correlation coefficient
 132 marks the improper nature of aggregate additive distortions. Given the non-uniform symbol probabilities
 133 and improper noise, we propose a maximum a posterior (MAP) detector for the optimal detection as
 134 opposed to the conventional minimum Euclidean or maximum likelihood (ML) detectors (Javed et al.
 135 (2021)). Thus, the detection criterion is given by

$$\hat{m}_{\text{PS}} = \arg \max_{1 \leq m \leq M} p_X(x_m) f_{Y_I, Y_Q|X}(y_I, y_Q|x_m), \quad (6)$$

136 where $f_{Y_I, Y_Q|X}(y_I, y_Q|x_m)$ is the conditional Gaussian probability density function (PDF) of y given x_m
 137 derived using (Javed et al., 2020, eq. 43)

$$f_{Y_I, Y_Q|X}(y_I, y_Q|x_m) = \frac{1}{2\pi\sigma_I\sigma_Q\sqrt{1-\rho_z^2}} \exp \left\{ \frac{-1}{2(1-\rho_z^2)} \left[\frac{(y_I - \sqrt{\alpha}\Re(x_m))^2}{\sigma_I^2} + \frac{(y_Q - \sqrt{\alpha}\Im(x_m))^2}{\sigma_Q^2} - \frac{2\rho_z(y_I - \sqrt{\alpha}\Re(x_m))(y_Q - \sqrt{\alpha}\Im(x_m))}{\sigma_I\sigma_Q} \right] \right\}. \quad (7)$$

3 ERROR PROBABILITY ANALYSIS

138 In this section, we derive the symbol error probability P_s for a system transmitting M -ary modulated
 139 symbols with prior probabilities p_m and subjected to HWD using the union bound and pairwise error

140 probability as follows:

$$P_s \leq \sum_{m=0}^{M-1} \sum_{n \neq m} P_m \Pr(x_m \rightarrow x_n | x_m). \quad (8)$$

141 The pairwise error probability, for receiving an erroneous symbol x_m given x_n was transmitted, can be
 142 derived using the following MAP rule,

$$\Pr(x_m \rightarrow x_n | x_m) = \Pr \left\{ P_m f_{Y_I, Y_Q}(y_I, y_Q | x_m) \leq P_n f_{Y_I, Y_Q}(y_I, y_Q | x_n) \right\}. \quad (9)$$

143 Using (7) and few simplifications, we get the following bit error probability

$$P_b \leq P_b^{\text{UB}}(M, \alpha, \mathbf{p}) \triangleq \frac{1}{\log_2(M)} \sum_{m=1}^M \sum_{\substack{n=1 \\ n \neq m}}^M p_m \mathcal{Q} \left(\beta_{mn}(\alpha) \ln \left(\frac{p_m}{p_n} \right) + \frac{1}{2\beta_{mn}(\alpha)} \right), \quad (10)$$

144 where β_{mn} is defined as

$$\beta_{mn}(\alpha) \triangleq \sqrt{\frac{\alpha^2 \left(\kappa^2 - \Re(\tilde{\kappa})^2 - \Im(\tilde{\kappa})^2 \right) + 2\alpha\kappa\sigma_w^2 + \sigma_w^4}{\alpha^2 \wp_{mn} + \alpha (\xi_{mnI}^2 + \xi_{mnQ}^2) 2\sigma_w^2}}, \quad (11)$$

145 with $\wp_{mn} = 2\xi_{mnI}^2(\kappa - \Re(\tilde{\kappa})) + 2\xi_{mnQ}^2(\kappa + \Re(\tilde{\kappa})) - 4\Im(\tilde{\kappa})\xi_{mnI}\xi_{mnQ}$ while $\xi_{mn} = d_{mn} = x_m - x_n$
 146 represents the distance between m^{th} and n^{th} symbols.

4 PROBLEM FORMULATION AND OPTIMIZATION

We propose a PS scheme, as a possible form of AS schemes, to effectively mitigate the drastic effects of additive distortions while transmitting with minimum power to minimize the EMF-exposure. To this end, we employ a higher-order M_{nu} probabilistically shaped quadrature amplitude modulation (QAM) offering more degrees of freedom and adaptive rates. The optimization problem targets the joint design of transmit power and symbol probabilities which minimize the exposure index while maintaining a throughput quality constraint. Assuming the set that includes all possible values of α as $\mathbb{A} = \{\alpha : 0 \leq \alpha \leq \alpha_{\text{max}}\}$, we can formulate the optimization problem as

$$\mathbf{P1} : \underset{\alpha \in \mathbb{A}, \mathbf{p} \in \mathbb{S}}{\text{minimize}} \quad \text{EI}(\alpha) \quad (12a)$$

$$\text{subject to} \quad \sum_{m=1}^{M_{\text{nu}}} |x_m|^2 p_m \leq 1, \quad (12b)$$

$$\left(1 - P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}) \right) H(\mathbf{p}) \geq \mathcal{T}_u, \quad (12c)$$

147 where $\mathcal{T}_u = (1 - P_b^{\text{UB}}(M_u, \alpha_{\text{max}}, \mathbf{p}_u)) \log_2(M_u)$ is the throughput of the uniformly distributed symbol
 148 constellation with maximum power transmission. Moreover, (12b) and (12c) represent the average power
 149 and throughput QoS constraints, respectively. In addition, $H(\mathbf{p})$ is the source entropy, which represents the

Algorithm 1 Alternate Optimization

- 1: **Initialize** $j \leftarrow 0, \epsilon \leftarrow \infty$ and **Set** tolerance δ
 - 2: **Choose** feasible starting points $\mathbf{p}^{(j)}$ and $\alpha^{(j)}$.
 - 3: **Evaluate** $\text{EI}(\alpha^{(j)})$.
 - 4: **while** $\epsilon \geq \delta$ **do**
 - 5: Solve **P1(a)** with given $\alpha^{(j)}$ and initial point $\mathbf{p}^{(j)}$ to obtain a feasible solution $\mathbf{p}^{(j^*)}$
 - 6: Solve **P1(b)** given $\mathbf{p}^{(j^*)}$ to obtain $\alpha^{(j^*)}$
 - 7: $\mathbf{p}^{(j+1)} \leftarrow \mathbf{p}^{(j^*)}$ and $\alpha^{(j+1)} \leftarrow \alpha^{(j^*)}$
 - 8: Evaluate $\text{EI}(\alpha^{(j+1)})$
 - 9: Update $\epsilon \leftarrow \left\| \text{EI}(\alpha^{(j+1)}) - \text{EI}(\alpha^{(j)}) \right\|$
 - 10: $j \leftarrow j + 1$
 - 11: **end while**
 - 12: **Solution parameters:** $\mathbf{p}^* \leftarrow \mathbf{p}^{(j+1)}$ and $\alpha^* \leftarrow \alpha^{(j+1)}$
 - 13: **Objective function:** $\text{EI}(\alpha^{(*)})$
-

150 transmitted rate in terms of bits per symbol per channel use and is defined as

$$H(\mathbf{p}) \triangleq \sum_{m=1}^{M_{\text{nu}}} -p_m \log_2(p_m). \tag{13}$$

The joint optimization problem is challenging due to the non-convex constraints. Therefore, we adopt an alternate optimization approach to iteratively solve the transmit power and symbol probabilities using the sub-problems (14) and (21), respectively. The alternate optimization Algorithm 1 begins with some initial feasible points $\mathbf{p}^{(j)}$ and $\alpha^{(j)}$ and evaluates $\text{EI}(\alpha^{(j)})$ for reference benchmark. Then, it finds a feasible solution of **P1(a)**, i.e., $\mathbf{p}^{(j^*)}$ that satisfies (12b) and (12c). Given a probability mass function (PMF) $\mathbf{p}^{(j^*)}$, we optimize **P1(b)** to minimize exposure index obtaining optimal $\alpha^{(j^*)}$, which are updated to attain initial points for the next iteration. This iterative process continues until reaching an acceptable tolerance δ . Consequently, the solution parameters yield a suitable PMF and transmission power, which render a minimum exposure index while **maintaining** a throughput QoS.

$$\mathbf{P1(a)} : \underset{\mathbf{p} \in \mathcal{S}}{\text{find}} \quad \mathbf{p} \tag{14a}$$

$$\text{which satisfies} \quad (12b) \text{ and} \tag{14b}$$

$$1 - P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}) \geq \mathcal{T}_u/H(\mathbf{p}), \tag{14c}$$

151 The problem **P1(a)** is a non-convex optimization problem due to the constraint (14c). Interestingly, $1/H(\mathbf{p})$
 152 is convex as the second derivative is always positive (see Appendix A). However, the bit error probability is
 153 a non-convex function in \mathbf{p} . Therefore, we tackle this challenge using the successive convex approximation
 154 approach based on the Taylor series approximation of the bit error probability. First-order Taylor series
 155 approximation of a function $f(x)$ around a point $x^{(k)}$ is given as

$$\tilde{f}(x, x^{(k)}) \approx f(x^{(k)}) + \nabla_x f(x^{(k)}) (x - x^{(k)}). \tag{15}$$

156 Thus, we need to compute $\nabla_{\mathbf{p}} P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p})$ and evaluate it at $\mathbf{p}^{(k)}$.

$$\nabla_{\mathbf{p}} P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}) = \begin{bmatrix} \frac{\partial P_b^{\text{UB}}}{\partial p_1} & \frac{\partial P_b^{\text{UB}}}{\partial p_2} & \cdots & \frac{\partial P_b^{\text{UB}}}{\partial p_{M_{\text{nu}}}} \end{bmatrix}. \quad (16)$$

157 In order to compute $\partial P_b^{\text{UB}} / \partial p_t$, we rewrite (10) as

$$P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}) = \frac{1}{\log_2(M_{\text{nu}})} \sum_{m=1}^{M_{\text{nu}}} \sum_{\substack{n=1 \\ n \neq m}}^{M_{\text{nu}}} p_m \int_{\Omega_{mn}}^{\infty} \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} du, \quad (17)$$

158 where Ω_{mn} is defined as,

$$\Omega_{mn} = \beta_{mn} \ln \left(\frac{p_m}{p_n} \right) + \frac{1}{2\beta_{mn}}. \quad (18)$$

159 Applying the Leibniz integral rule on (17) yields the following partial derivative

$$\frac{\partial P_b^{\text{UB}}}{\partial p_t} \leq \frac{1}{\log_2(M_{\text{nu}})} \sum_{\substack{n=1, \\ n \neq t, \\ m=t}}^{M_{\text{nu}}} \left(\mathcal{Q}(\Omega_{mn}) - \frac{\beta_{mn}}{\sqrt{2\pi}} e^{-\frac{\Omega_{mn}^2}{2}} \right) + \frac{1}{\log_2(M_{\text{nu}})} \sum_{\substack{m=1, \\ m \neq t, \\ n=t}}^{M_{\text{nu}}} \frac{\beta_{mn} p_m}{\sqrt{2\pi} p_n} e^{-\frac{\Omega_{mn}^2}{2}}. \quad (19)$$

160 Now, $P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p})$ can be approximated from (15), (16), and (19) using first-order Taylor series
 161 expansion around an initial probability vector $\mathbf{p}^{(k)}$ as

$$\tilde{P}_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}, \mathbf{p}^{(k)}) \triangleq P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}^{(k)}) + \nabla_{\mathbf{p}} P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}^{(k)}) (\mathbf{p} - \mathbf{p}^{(k)}). \quad (20)$$

Conclusively, we can solve **P1(a)** by replacing $P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p})$ in (14c) with its Taylor series approximation $\tilde{P}_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}, \mathbf{p}^{(k)})$ and solving the resultant convex feasibility problem iteratively using the well known successive convex approximation approach (Liu et al. (2019)). On the other hand, we solve **P1(b)** for a given PMF to obtain an optimal α^* which minimizes the EMF exposure.

$$\mathbf{P1(b)} : \underset{\alpha \in \mathbb{A}}{\text{minimize}} \quad \text{EI}(\alpha) \quad (21a)$$

$$\text{subject to} \quad (12c) \quad (21b)$$

162 Intuitively, the throughput of probabilistically shaped M_{nu} -ary QAM is an increasing function of α because
 163 $P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p})$ is a decreasing function of α . Therefore, the solution to problem **P1(b)** is simply obtained
 164 by solving the inequality constraint with equality as

$$P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}) = 1 - \mathcal{T}_u/H(\mathbf{p}) \quad (22)$$

165 The root of the non-linear equation $\Upsilon(\alpha) = P_b^{\text{UB}}(M_{\text{nu}}, \alpha, \mathbf{p}) + \mathcal{T}_u/H(\mathbf{p}) - 1$ can be obtained using
 166 Newton-Raphson method, which begins with an initial guess $\alpha^{(j)}$ and updates it with every iteration as
 167 (Kelley (2003)),

$$\alpha^{(j+1)} = \alpha^{(j)} - \frac{\Upsilon(\alpha^{(j)})}{\Upsilon'(\alpha^{(j)})}. \quad (23)$$

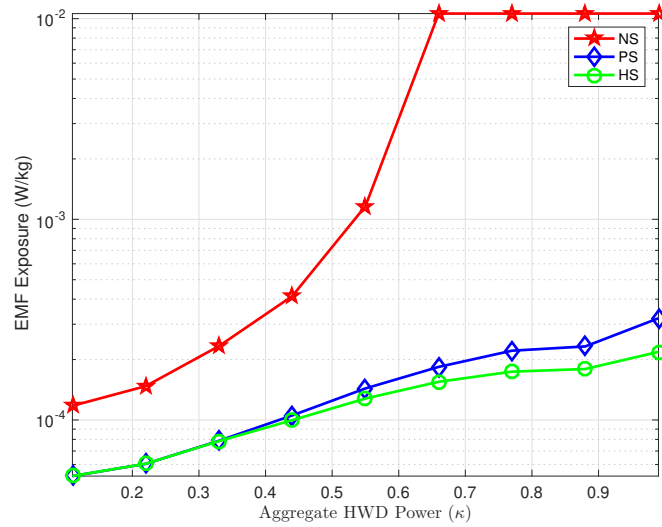


Figure 2. Exposure Index [W/kg] versus HWD power when NS transmits with α_{\max} .

168 It is worthy to note that $\Upsilon'(\alpha^{(j)}) \neq 0$ and is computed from (17) as

$$\Upsilon'(\alpha) = \frac{1}{\log_2(M_{\text{nu}})} \sum_{m=1}^{M_{\text{nu}}} \sum_{\substack{n=1 \\ n \neq m}}^{M_{\text{nu}}} p_m \frac{e^{-\Omega_{mn}^2/2}}{\sqrt{2\pi}} \left(\frac{1}{2\beta_{mn}^2} - \ln \left(\frac{p_m}{p_n} \right) \right) \frac{\partial \beta_{mn}}{\partial \alpha}, \quad (24)$$

where $\frac{\partial \beta_{mn}}{\partial \alpha}$ is expressed as

$$\frac{\partial \beta_{mn}}{\partial \alpha} = \frac{1}{2} \sqrt{\frac{\alpha^2 \wp_{mn} + \alpha (\xi_{mnI}^2 + \xi_{mnQ}^2) 2\sigma_w^2}{\alpha^2 (\kappa^2 - \Re(\tilde{\kappa})^2 - \Im(\tilde{\kappa})^2) + 2\alpha\kappa\sigma_w^2 + \sigma_w^4}} \times \quad (25)$$

$$\left\{ \frac{2\alpha (\kappa^2 - \Re(\tilde{\kappa})^2 - \Im(\tilde{\kappa})^2) + 2\kappa\sigma_w^2}{\alpha^2 \wp_{mn} + \alpha (\xi_{mnI}^2 + \xi_{mnQ}^2) 2\sigma_w^2} - \frac{\psi(2\alpha\wp_{mn} + (\xi_{mnI}^2 + \xi_{mnQ}^2) 2\sigma_w^2)}{(\alpha^2 \wp_{mn} + \alpha (\xi_{mnI}^2 + \xi_{mnQ}^2) 2\sigma_w^2)^2} \right\} \quad (26)$$

169 with $\psi = \alpha^2 (\kappa^2 - \Re(\tilde{\kappa})^2 - \Im(\tilde{\kappa})^2) + 2\alpha\kappa\sigma_w^2 + \sigma_w^4$. The process repeats until the desired criterion is
 170 met in terms of precision, i.e., $\Upsilon(\alpha)$ becomes acceptably small, the change in α is lesser than the predefined
 171 limit or maximum number of iterations.

5 NUMERICAL RESULTS

172 In this section, we present some numerical results to quantify the EMF exposure caused by the proposed PS
 173 scheme instead of the conventional no-shaping (NS). We consider an indoor active user in next-generation
 174 cellular network suffering from improper HWDs. We assume $\kappa \in \{0, 1\}$, $\rho = 0.9$, $\sigma_w^2 = 1$, $M_u = 8$ -QAM
 175 and $M_{\text{nu}} = 16$ -QAM unless specified otherwise. The reference whole-body SAR values are taken from
 176 (Vermeeren et al. (2015a), Table 27), which were computed using 3D EM simulation platforms based on
 177 the finite difference time domain and finite integration technique method. For instance, whole body SAR^{ref}
 178 for standing adult is 0.0053 while for sitting is 0.0047. On the other hand, for child it is 0.015 and 0.014,

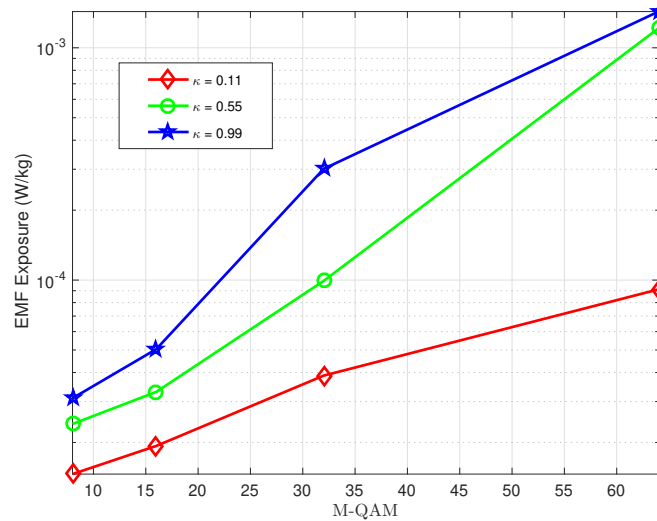


Figure 3. Exposure Index [W/kg] versus modulation order

179 respectively, for the uplink voice communication at 2.6 GHz frequency band. In number results, we take
 180 the average reference SAR weighted by the number of users in each category along with their respective
 181 postures. In particular, we consider $SAR^{ref} = 41 \times 10^{-4}$ W/Kg for data users and $SAR^{ref} = 63 \times 10^{-4}$ W/Kg
 182 for voice users per unit power (Ibraiwish et al. (2022)). Evidently, the SAR reference value of voice users
 183 is higher as compared to the data users because the voice users are expected to keep the cellular mobile
 184 phones near their brains.

At first, we investigate the EMF exposure of a single active user for a range of HWD levels. We present the comparison between the traditional NS scheme employing M_u -QAM and the proposed PS scheme employing M_{nu} -QAM. For a fair comparison, we minimize the exposure index of M_u -QAM with uniform distribution \mathbf{p}_u (NS) scheme to ensure a throughput threshold of $\mathcal{T}_0 = 2.997$ bits/sec, i.e.,

$$\mathbf{P2(a)} : \underset{\alpha_u \in \mathbb{A}}{\text{minimize}} \quad EI(\alpha_u) \tag{27a}$$

$$\text{subject to} \quad \left(1 - P_b^{UB}(M_u, \alpha, \mathbf{p}_u)\right) \log_2(M_u) \geq \mathcal{T}_0, \tag{27b}$$

185 The simulation results provide the insights of the Exposure Index of no-shaping versus probabilistic
 186 shaping for a range of HWD to achieve a target throughput, as shown in Figure 2. Evidently, they reveal the
 187 superiority of employing PS to successfully limit the EMF exposure while maintaining the QoS in terms of
 188 throughput. The advantage of PS over NS is particularly prominent for higher distortion levels. Moreover,
 189 we move a step further to investigate the performance of the HS scheme with M_{nu} -QAM employing an
 190 aggregate of GS and PS as detailed in Javed et al. (2021). Interestingly, HS reduces the EMF exposure
 191 for higher distortion levels but the gain is insignificant given the added complexity in designing GS and
 192 PS parameters. Conclusively, PS is the preferred choice as it can reduce EMF exposure up to 98% with
 193 affordable complexity.

194 Next, we study the impact of varying the modulation order for three different distortion levels on the
 195 EMF exposure in Figure 3. We assumed modulation order M_{nu} ranging from 8-QAM to 64-QAM for the
 196 proposed PS. Noticeably, the increasing modulation order increases the EMF exposure for all HWD levels,

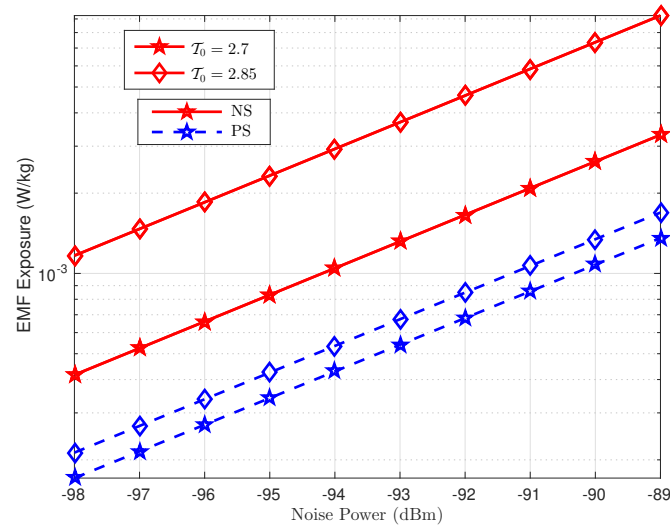


Figure 4. Exposure Index [W/kg] versus noise variance.

197 i.e., least, medium and high distortion levels, however, at different paces. Intuitively, more transmit power
 198 is required to meet the QoS constraint in a highly distorted system instead of the least distorted system,
 199 which advocates a considerable EMF exposure of higher distortion levels.

200 Similarly, we analyzed the benefits of PS over NS for a range of noise variance (-98dBm to -89dBm)
 201 for two different target throughputs, i.e., $\mathcal{T}_0 = 2.7$ bits/sec and $\mathcal{T}_0 = 2.85$ bits/sec. We can observe a 55%
 202 and 81.82% percent reduction in EMF exposure at -98dBm for the target threshold rates of 2.7 and 2.85
 203 bits per sec per channel, respectively, with PS over NS as shown in Figure 4. Intuitively, PS outperforms
 204 NS for the entire range of noise variance in decreasing the EMF exposure on a user.

6 CONCLUSION

205 Throughout this paper, we highlight the significance of employing probabilistic shaping (PS) to mitigate
 206 the drastic effects of improper hardware distortions and effectively reduce the user's EMF exposure while
 207 maintaining a target threshold. The numerical results reveal up to a 98% percentage reduction in Exposure
 208 Index with the help of PS as compared to the conventional NS. Further investigation demonstrates a minor
 209 gain with HS over PS with significant added complexity. Thus, we conclude that mere PS is the preferred
 210 choice to reduce the exposure index, given a trade-off between lowering EI and increasing computational
 211 complexity.

DATA AVAILABILITY STATEMENT

212 The original contributions of this work are presented in the article/Supplementary Material, further inquiries
 213 can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

214 The study was conducted as a collaboration among all authors. AE conceived the work. The manuscript
 215 was mainly drafted by SJ and was revised and approved by all co-authors.

APPENDIX A: PROOF OF CONVEXITY

216 The first and second order derivatives of $1/H(\mathbf{p})$ are given as

$$\nabla_{\mathbf{p}} (1/H(\mathbf{p})) = -H'(\mathbf{p}) / [H(\mathbf{p})]^2 \quad (28)$$

217

$$\nabla_{\mathbf{p}}^2 (1/H(\mathbf{p})) = -\frac{[H(\mathbf{p})]^2 H''(\mathbf{p}) - 2 [H'(\mathbf{p})]^2 H(\mathbf{p})}{[H(\mathbf{p})]^4} \geq 0 \quad \because \quad H(\mathbf{p}) \geq 0 \text{ and } H''(\mathbf{p}) \leq 0 \quad (29)$$

218 The second order derivative is always positive given the non-negative and concave nature of information
219 entropy. Hence, we can safely conclude that $1/H(\mathbf{p})$ is a convex function in \mathbf{p} .

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