

On Energy Efficiency of Prioritized IoT Systems

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Abstract—The inevitable deployment of 5G and the Internet-of-Things (IoT) sheds the light on the importance of the energy efficiency (EE) performance of Device-to-Device (DD) communication systems. In this work, we address a potential IoT application, where different prioritized DD system, i.e., Low-Priority (LP) and High-Priority (HP) systems, co-exist and share the spectrum. We maximize the EE of each system by proposing two schemes. The first scheme optimizes the individual transmission power and the spatial density of each system. The second scheme optimizes the transmission power ratio of both systems and the spatial density of each one. We also construct and analytically solve a multi-objective optimization problem that combines and jointly maximizes both HP and LP EE performance. Unique structures of the addressed problems are verified. Via numerical results we show that the system which dominates the overall EE (combined EEs of both HP and LP) is the system corresponding to the lowest power for low/high power ratio (between HP and LP systems). However, if the power ratio is close to one, the dominating EE corresponds to the system with higher weight.

Index Terms—Spectrum sharing, Energy efficiency, Resource allocation, Spatial Randomness, Device-to-Device,

I. INTRODUCTION

High throughput data communication and reliable (long lifetime and large capacity) sensor, DD, and cellular networks are essential for the realization of 5G and Internet-of-Things (IoT). These emerging technologies result in significant power consumption requirements. Therefore, energy measurement metrics are essential for realizing future technologies and guaranteeing a certain energy efficiency (EE) performance. Researchers have been thoroughly investigating the greenness of communication networks. Variety of EE metrics have been proposed based on different factors and several communication layers [1]–[4]. To capture the spatial randomness of such networks, e.g., IoT, an interesting tool, i.e., stochastic geometry, showed promising techniques which can be utilized to provide tractable performance expressions [5].

Several works have utilized stochastic geometric tools to tackle the outage probability and spatial capacity of communication networks [6]–[10]. However, working on utilizing this tool in improving the EE is still an open research problem. The authors of [6] have tackled the EE performance of a heterogeneous network. Fairness of the sub-channel allocation has also been addressed. On the other hand, authors of [7] have addressed the EE maximization of several sharing systems from a game theory perspective. They considered non-cooperative game between different systems, with and without incorporating pricing to the power control game. In a cellular system, authors of [8] have modeled a system that contains both macro basestations (MBSs) and femto-cell access points (FAPs) and their associated user equipments (UEs) as two

independent Poisson point processes. They do not consider the interference between different systems, unlike our work which considers the inter/intra interference. The maximum EE is addressed, with respect to (w.r.t.) sub-channels allocation, while considering the impact of different diversity schemes. The authors of [9] modeled the two-tier cellular system’s components, i.e., MBSs, small-cell access points (SAPs), and UEs as independent Poisson point process (PPP). They analyzed the data rate associated with different communication scenarios, i.e., MBS-to-UE, SAP-to-UE, and MBS-to-SAP. Also, the data rate to system’s power ratio is obtained.

In this work, we consider an IoT environment where different priorities systems, operating with multi-users on the same spectrum, might co-exist. For instance, in industrial IoT, a factory might have several connected machines that contributes to the main factory operations, while other machines, such as temperature sensors etc., might have less operational priority (such as, accepting different outage priority, transmitting power, EE values). Motivated by the aforementioned IoT scenarios, we consider High-Priority system (HPS) and Low-Priority system (LPS) co-exist in the same area while sharing the spectrum. We capture the spatial randomness of the nodes by utilizing the stochastic geometric approach. The targeted performance metric of all systems is the EE, defined as, the transmission capacity (successful transmissions per unit area) to power ratio. We enforce a High-Priority system (HPS)’s outage constraint on Low-Priority user (LPU) transmission to guarantee minimum HPS’s quality of service (QoS).

Our contributions are summarized as follows. We maximize the individual EE of both Low-Priority system (LPS) and HPS. Maximum EE is achieved via proposing two schemes, i.e., scheme 1 and scheme 2. Optimal transmission power and spatial density of each system are derived in scheme 1. The individual strict pseudo-concave structure of the problem is verified for scheme 1. Whereas, Optimal transmission power ratio of both systems and spatial density of each system are derived in scheme 2. In this scheme, the joint strict pseudo-concave and quasi-concave structures of the associated problem have been verified. We then construct a combined multi-objective EE as a weighted sum of both LPU and High-Priority user (HPU) systems. In this approach, we maximize the combined EE w.r.t. LPU’s and HPU’s transmission power and spatial density. The global joint optimal solution set is obtained via verifying the mathematical structure of the multi-objective optimization problem and proposing an alternating algorithm, while satisfying the necessary and sufficient conditions. Numerical results showed that if the LP-to-HP power ratio is high or low ($\gg 1$ or $\ll 1$), then the dominating

EE is the one corresponding to the lower power. However, if LP-to-HP power ratio is close to one, then the dominating EE is the one with higher weight.

In the following, we show the distinction points between our proposed model and solutions and the existing ones. Unlike our work, the work of [6] did not derive the optimal power allocation of the associated problem. Also, the authors of [7] did not provide analytical results to both nodes' spatial density and transmission power. They also did not combine EE of several sharing systems. The problem models of [8], [9] are based on a master slave communication system, unlike our prioritized DD in-band sharing system. Also, different optimization variables and solution methodologies are considered inhere. We also do not ignore the intra/inter interference of different communication pairs, as the case of [9]. Unlike our model, the formulation of EE metric in [9], [10] does not maximizes the combination of two sharing systems' EE.

II. SYSTEM MODEL AND TRANSMISSION CAPACITY

Targeting EE of spectrum sharing networks requires obtaining each system's capacity. In this section, we describe the system model and transmission capacity of each system.

A. Spectrum Sharing Network Model

In this network model, we consider two multi-user DD systems, with different priorities, that co-exist in the same area. Each pair of nodes (transmitter and receiver) belongs to either LPS or HPS network's transceivers. The set of all transmitter nodes, i.e., $\{l, h\} \in \Phi$, follows PPP distribution, where the nodes are uniformly distributed in the captured area. This distribution is well accepted in the literature because of the randomness of the nodes locations, i.e., this distribution represents the worst-case scenario among other distributions, e.g., [11]. The mean of the PPP distributed nodes is λ_q , where $q \in \{l, h\}$. This is interpreted as the average number of nodes in a unit area is equal to λ_q . Since different receiving nodes have similar received signal statistics, we therefore conduct the analysis on a receiving node at the origin of the map. The interfering nodes follow a marked Poisson point process (MPPP), i.e., transmitting nodes of a system q are expressed as $\Xi_q = \{(\mathcal{X}_{qj}, h_{qj})\}$. The location of the transmitter is noted as \mathcal{X}_{qj} and the channel between the transmitter and the receiving node (at the origin) is h_{qj} . This channel follows a Rayleigh distribution with a unity mean and its modulus squared is expressed as $|h_{qj}|^2 = \gamma_{qj}$. We also assume that all the transmission nodes in each priority system transmit with the same adaptive power, i.e., P_q for $q \in \{l, h\}$. The received power at the desired receiver is expressed as $|x_q|^{-\alpha} P_q \gamma_{qj}$ for $q \in \Phi$, where $|x_q|$ is the distance to the receiving node at the origin and α is the pathloss exponent. Note that we omit the j at the studied receiving node since it is at the origin.

B. Success Probability and Transmission Capacity

Inhere, we define the successful transmission probability of sharing network and the associated transmission capacity.

In order to express the targeted system's signal to noise and interference ratio (SINR), noted as system t , we consider all

interferences from the same system, t , and the other system, q . The SINR of the targeted system is expressed as follows,

$$\text{SINR}_t = \frac{|x_t|^{-\alpha} P_t \gamma_{ti}}{\sum_{q \in \Phi} \sum_{x_q \in \Xi_q} |x_q|^{-\alpha} P_q \gamma_{qj} + \mathcal{N}_0}. \quad (1)$$

We ignore the effect of the thermal noise, since we assume the interference from all the nodes has larger effect on the desired signal compared to the thermal noise. Therefore, we consider the targeted signal to interference ratio (SIR). Hence, the successful transmission probability is defined as follows,

$$\mathbb{P}_t = \Pr \left\{ \Theta_t \geq \theta_t \right\} = \Pr \left\{ \underbrace{\frac{|x_t|^{-\alpha} P_t \gamma_{ti}}{\sum_{q \in \Phi} \sum_{x_q \in \Xi_q} |x_q|^{-\alpha} P_q \gamma_{qj}}}_{\mathcal{I}_t} \geq \theta_t \right\}, \quad (2)$$

where θ_t is the desired SIR. Note that the interference is a sum of measurable functions of a MPPP. It follows that after applying several known techniques (i.e., Laplace transformation, Campbell's Theorem, random variable conditioning, and change of variable) on (2), the successful transmission probability of LPS is found as [5],

$$\mathbb{P}_l = \exp \left(-\eta'_l \left(\lambda_l + \lambda_h (P_h/P_l)^{2/\alpha} \right) \right), \quad (3)$$

where $\eta'_l = \pi R_t^2 \theta_t^{\frac{2}{\alpha}}$. The successful transmission probability of the HPS is found in similar lines with that in the LPS while switching the sub-script of the parameter from x_l to x_h , where x is any symbol used interchangeably for both LPS and HPS.

The transmission capacity of each system in the sharing area must be obtained to express the EE metric. The corresponding LP's transmission capacity is expressed as follows [5],

$$\mathcal{T}_l = \lambda_l \exp \left(-\eta'_l \left(\lambda_l + \lambda_h (P_h/P_l)^{2/\alpha} \right) \right), \quad (4)$$

The HP's transmission capacity is also find in similar lines with (4) while changing the sub-script because of the symmetry between both expressions.

III. ACHIEVABLE ENERGY EFFICIENCY OF LPU AND HPU

In this section, we formulate EE of LPU's network and derive the optimal solution, while protecting the HPU network. Following similar steps, we derive the optimal solution of EE performance of HPU network.

The EE of LPU's network is defined as the ratio of transmission capacity to power times the node density,

$$\mathcal{E}_l(P_l, \lambda_l, P_h, \lambda_h) = \lambda_l \frac{e^{(-\eta'_l (\lambda_l + \lambda_h (P_h/P_l)^{2/\alpha}))}}{(k_t P_l + k_c)}, \quad (5)$$

where k_c and k_t are assigned parameters which correspond to circuit and power amplifier constant power consumption.

The EE problem, formulated below, of LPU's system maximizes LPU's EE metric subject to maintaining a certain LPU's and HPU's outage probabilities,

$$\max \quad \lambda_l e^{(-\eta'_l (\lambda_l + \lambda_h (P_h/P_l)^{2/\alpha}))} / (k_t P_l + k_c) \quad (6a)$$

$$\text{s.t.} \quad \mathfrak{C}_1: 1 - e^{(-\eta'_l (\lambda_l + \lambda_h (P_h/P_l)^{2/\alpha}))} \leq \epsilon_l \quad (6b)$$

$$\mathfrak{C}_2: 1 - e^{(-\eta'_h (\lambda_h + \lambda_l (P_l/P_h)^{2/\alpha}))} \leq \epsilon_h, \quad (6c)$$

where ϵ_l and ϵ_h are the outage probabilities tolerance thresholds. We propose two schemes to solve problem (6). The main difference between the two schemes is the definition of the optimization variables.

1) *Scheme 1*: In this scheme, we optimize LPU's transmission power, P_l , and LPU's spatial density, λ_l .

We begin by converting the probabilistic constraints \mathcal{C}_1 and \mathcal{C}_2 into an instantaneous constraint. These constraints act as boundaries for each of the optimization variables, i.e., P_l and λ_l . Constraint \mathcal{C}_1 upper bounds P_l as follows,

$$P_l \leq \left[\frac{\log(1 - \epsilon_h)^{-1}}{\lambda_l \eta'_h} - \frac{\lambda_h}{\lambda_l} \right]^{\frac{\alpha}{2}} P_h = P_s^+. \quad (7)$$

Constraint \mathcal{C}_2 lower bounds P_l as follows,

$$P_l \geq \left[\frac{\log(1 - \epsilon_l)^{-1}}{\lambda_h \eta'_l} - \frac{\lambda_l}{\lambda_h} \right]^{\frac{-\alpha}{2}} P_h = P_s^-. \quad (8)$$

Problem (6) w.r.t. P_l is difficult to solve by conventional method. That is due to the fractional, exponential, and geometrical nature of P_l in (6a). In order to find the optimal P_l which maximizes (6) we apply geometric optimization techniques and change of variable, on variable P_l such that, $P_n = (P_l)^{\frac{2}{\alpha}}$ and $P_l = (P_n)^{\frac{\alpha}{2}}$. Since the new variable is a monotone function over the range of $P_l \geq 0$, we know from [12] that optimizing problem (6) w.r.t. P_n is equivalent to optimizing it w.r.t. P_l . We apply transformation technique to the objective function in (6a), i.e., $\hat{\mathcal{E}}_l = f_0(\mathcal{E}_l) = \log(\mathcal{E}_l)$. It is proven in [12] that if $f_0(\cdot)$ is $\mathbb{R} \rightarrow \mathbb{R}$ and monotonic increasing function, then maximizing \mathcal{E}_l is equivalent to maximizing $\hat{\mathcal{E}}_l = f_0(\mathcal{E}_l)$. It is observed that these conditions are satisfied by the logarithmic function. Both constraints \mathcal{C}_1 and \mathcal{C}_2 are peak constraints on P_l , thus we apply them after obtaining the optimal P_l . Problem (6) is equivalent into the following problem,

$$\max_{P_s^- \leq P_n \leq P_s^+} \log \left[\lambda_l \exp \left[-\eta'_l \left[\lambda_l + \lambda_h P_h^{\frac{2}{\alpha}} / P_n \right] \right] \right] / \left[k_t P_n^{\frac{\alpha}{2}} \right]. \quad (9)$$

The solution of (9) is summarized in the following lemma.

Lemma 1. *The optimal LPU's transmission power that maximizes problem (9) is expressed as follows,*

$$\hat{P}_l = \min \left\{ \max \{ P_l^*, P_s^- \}, P_s^+ \right\}, \quad (10)$$

where P_l^* is expressed as follows,

$$P_l^* = \left(\frac{2\eta'_l \lambda_h}{\alpha} \right)^{\frac{\alpha}{2}} P_h. \quad (11)$$

Proof. The proof is given in our full version work [13]. \square

It is observed that the optimal LPU's transmission power increases with HPU transmission power and spatial density.

We now find the optimal LPU's spatial density which maximizes problem (6). We begin by transforming constraints \mathcal{C}_1 and \mathcal{C}_2 as upper bounds to the spatial density. Let us note the overall upper bound as follows,

$$\lambda_s^+ = \min \left(\lambda_{s1}^+, \lambda_{s2}^+ \right), \quad (12)$$

where

$$\lambda_{s1}^+ = \left[\frac{\log(1 - \epsilon_h)^{-1}}{\eta'_h} - \lambda_h \right] \left(\frac{P_h}{P_l} \right)^{2/\alpha}, \quad (13)$$

and

$$\lambda_{s2}^+ = \left[\frac{\log(1 - \epsilon_l)^{-1}}{\eta'_l} - \lambda_h \left(\frac{P_h}{P_l} \right)^{2/\alpha} \right]. \quad (14)$$

We then rewrite problem (6) as follows,

$$\max_{\lambda_l \leq \lambda_s^+} \lambda_l \exp \left(-\eta'_l \left(\lambda_l + \lambda_h (P_h/P_l)^{2/\alpha} \right) \right) / k_t P_l. \quad (15)$$

The solution of (15) is summarized in the following lemma.

Lemma 2. *The optimal LPU's spatial density that maximizes problem (15) is expressed as follows,*

$$\hat{\lambda}_l = \min \left\{ \lambda_s^+, \frac{1}{\eta'_l} \right\}. \quad (16)$$

Proof. The proof is given in our full version work [13]. \square

Note that the optimal LPU's spatial density decreases with the increase of the distance between nodes and the increase of the SIR threshold.

Note that in order to get the optimal expressions of transmission power and nodes spatial density for the HP system, we face two choices. One is to protect the LP users transmission via similar constraint to that in \mathcal{C}_2 in (6) but from HP transmission perspective. In this case, both the optimal power and user density, are found in similar lines with the ones for LPS while changing the sub-script. The other choice is not to protect the LP's transmission. Hence, the upper bound on the optimal HP's transmission power is discarded. Also, one of the two upper bounds on the HPU's spatial density is discarded, as HPS does not care about protecting LPS's performance. Otherwise, all derivation steps are similar to that of LPS case.

2) *Scheme 2*: In this scheme, we maximize problem (6) by jointly optimizing the transmission power ratio, i.e., $\zeta_n = \frac{P_h}{P_l}$, and LPU's spatial density, i.e., λ_l . The main advantage of this scheme over scheme 1 is that the joint optimal solution is guaranteed without the necessity to use alternating algorithm.

After substituting $\zeta_s = \left(\frac{P_h}{P_l} \right)^{\frac{2}{\alpha}}$ in problem (6), the targeted optimization problem is rewritten as follows,

$$\max_{\zeta_s, \lambda_l} \lambda_l \exp \left(-\eta'_l (\lambda_l + \lambda_h \zeta_s) \right) / \left(k_t P_h \zeta_s^{\frac{-\alpha}{2}} \right) \quad (17a)$$

$$\text{s.t.} \quad \mathcal{C}_1: \lambda_l + \lambda_h \zeta_s + \frac{1}{\eta'_l} \log(1 - \epsilon_l) \leq 0 \quad (17b)$$

$$\mathcal{C}_2: \lambda_l + \zeta_s \left(\lambda_h + \frac{1}{\eta'_h} \log(1 - \epsilon_h) \right) \leq 0 \quad (17c)$$

The solution of (17) is summarized as follows.

Theorem 1. *The global joint optimal solution set of problem (17) is obtained as follows.*

The optimal transmission power ratio is expressed as,

$$\hat{\zeta}_s = \frac{2}{\alpha} \left[\lambda_h (\eta'_s + \lambda_1 + \lambda_2) + \frac{\lambda_2}{\eta'_p} \log(1 - \epsilon_p) \right]^{-1}. \quad (18)$$

The optimal LPU's spatial density is expressed as,

$$\hat{\lambda}_l = (\eta'_s + \lambda_1 + \lambda_2)^{-1}. \quad (19)$$

The Lagrangian multipliers λ_1 and λ_2 are obtained by solving the Karush-Kuhn-Tucker (KKT) conditions associated with \mathcal{C}_1 and \mathcal{C}_2 , respectively.

Proof. The proof is given in our full version work [13]. \square

Remark 1. It is interesting to note that both solutions of problem (6), under scheme 1, and problem (17), under scheme 2, are equivalent, under some scenarios. Note that under inactive constraints, i.e., $\lambda_1 = 0$ and $\lambda_2 = 0$, the LPU's transmission power, obtained from the optimal ζ_s in (18), is equal to P_l^* in (11). Similarly, under inactive constraints we note that λ_l in (19) is equivalent to that in (16).

To maximize HPU's EE utilizing scheme 2, we use similar analogue to that of scheme 1.

IV. LINEARLY COMBINED EE LPU AND HPU SYSTEMS

In this section, we maximize a weighted sum of \mathcal{E}_l and \mathcal{E}_h w.r.t. transmission power of both LPU and HPU systems in addition to the spatial density of both LPU and HPU system. This problem is extremely complicated to solve, because the joint concavity or quasi-concavity structure of the objective function w.r.t. all optimization variables cannot be verified. Therefore, we solve this problem by individually maximizing it w.r.t. each optimization variable. We then find the optimal expression for each variable. Finally, we propose an iterative algorithm, which utilizes the individual optimality of each variable to guarantee a global joint optimal solution of the problem.

The formulation of the weighted sum total EE metric problem is expressed as follows,

$$\max \quad \mathcal{E}_{TS} = \alpha_l \mathcal{E}_l(P_l, P_h, \lambda_l, \lambda_h) + \alpha_h \mathcal{E}_h(P_l, P_h, \lambda_l, \lambda_h) \quad (20a)$$

$$\text{s.t.} \quad \mathcal{C}_1; \quad \mathcal{C}_2; \quad (20b)$$

where α_s and α_p are the weighting parameters of each metric. It is difficult to find a joint structure, i.e., concavity or pseudo-concavity, of problem (20) w.r.t. all optimization variables. Therefore, we begin our solution by maximizing problem (20) w.r.t. each variable separately, i.e., P_l , λ_l , P_h , and λ_h .

The optimal P_l which maximizes problem (20) cannot be obtained in a similar way to the optimal P_l which maximizes (6). This is due to the existence of P_l in the exponential term of \mathcal{E}_h . Furthermore, it is not possible to, analytically, find the zeros of the first derivative of Lagrangian function to solve (20) w.r.t. P_l . That is because there is a weighted sum of exponential terms and each one includes a different function of P_l . To overcome this problem, we introduce a new variable, i.e., P_{sp} . This variable replaces P_l in HPU's \mathcal{E}_h term, i.e., $\mathcal{E}_h = \frac{\lambda_h \exp(-\eta'_h (\lambda_h + \lambda_l (P_{sp}/P_h)^{2/\alpha}))}{(k_t P_h + k_c)}$. A new constraint must be added to link both variables, i.e., $P_l = P_{sp}$. We then apply the change of variables to both P_l and P_{sp} , such that $P_n = P_l^{2/\alpha}$ and $P_{np} = P_{sp}^{2/\alpha}$. The maximization problem of (20) is rewritten as follows,

$$\max_{(P_s^-)^{2/\alpha} \leq \{P_n, P_{np}\} \leq (P_s^+)^{2/\alpha}} \quad \alpha_l \mathcal{E}_l(P_n) + \alpha_h \mathcal{E}_h(P_{np}) \quad (21a)$$

$$\text{s.t.} \quad P_n = P_{np} \quad (21b)$$

Remark 2. Note that the second term of (21a) is constant w.r.t. P_n , i.e., $\frac{\alpha_h \lambda_h \exp(-\eta'_h (\lambda_h + \lambda_l P_{np}/P_h^{2/\alpha}))}{(k_t P_h + k_c)}$ is not a function of P_n . Whereas the first term in (21a) is constant w.r.t. P_{np} , i.e., $\frac{\lambda_l \exp(-\eta'_l (\lambda_l + \lambda_h P_h^{2/\alpha}/P_n))}{k_t P_n^{\alpha/2} + k_c}$ is not a function of P_{np} .

Utilizing Remark 2 and the geometric optimization techniques, which are used in Sec. III, we obtain the optimal value of P_l in the following proposition.

Proposition 1. The optimal value of P_n that maximizes (20), given P_h and λ_h , is expressed as follows,

$$\hat{P}_n = \min\{\max\{P_n^o, (P_s^-)^{2/\alpha}\}, (P_s^+)^{2/\alpha}\} \quad (22)$$

where

$$P_n^o = \left[\alpha + \sqrt{\alpha^2 - 16\mu\eta'_s \lambda_h P_h^{2/\alpha} / 4\mu} \right] \quad (23)$$

P_s^+ and P_s^- are expressed in (7) and (8), respectively. Whereas, the optimal value of P_{np} that maximizes (20), given P_h , λ_l , and λ_h , is expressed as follows,

$$\hat{P}_{np} = \min\{\max\{P_{np}^o, (P_s^-)^{2/\alpha}\}, (P_s^+)^{2/\alpha}\} \quad (24)$$

where

$$P_{np}^o = \frac{P_h^{2/\alpha}}{\eta'_p \lambda_l} \log \left[\frac{\eta'_p \lambda_l \lambda_h \alpha_p e^{-\eta'_p \lambda_h}}{\mu k_t P_h^{2/\alpha+1}} \right] \quad (25)$$

The parameter μ , in both (23) and (25), is the Lagrangian multiplier associated with the equality constraint in (21b). The value of μ is obtained by finding the zeros of the following function.

$$g(\mu) = \left[2w\mu - \frac{\mu P_h^{2/\alpha}}{\eta'_h \lambda_l} \log(\mu) - \frac{\alpha}{2} \right]^2 - \frac{\alpha^2}{4} + 4\mu\eta'_l \lambda_h P_h^{2/\alpha} = 0 \quad (26)$$

where $w = \frac{P_h^{2/\alpha}}{\eta'_p \lambda_l} \left[\log \left(\eta'_p \lambda_l \alpha_p \lambda_h e^{-\eta'_p \lambda_h} \right) - \log \left(k_t P_h^{1+2/\alpha} \right) \right]$.

Proof. The proof is given in Appendix A. \square

We now solve for the value of λ_l that maximizes problem (20). Utilizing similar geometric optimization techniques that are used in Sec. III and after some algebraic manipulations we find that maximizing problem (20) w.r.t. λ_l is equivalent to the following maximization problem.

$$\max_{\lambda_l \leq \lambda_s^+} \log \left(e^{-\eta'_l \lambda_l} (a_s \lambda_l + c_s e^{-d_s \lambda_l}) \right) \quad (27)$$

where $a_s = \frac{\alpha_l}{k_t P_l} \exp \left(-\eta'_l \lambda_h \frac{P_h^{2/\alpha}}{P_l^{2/\alpha}} \right)$, $c_s = \frac{\alpha_h}{k_t P_h} \exp(-\eta'_h \lambda_h)$,

and $d_s = \eta'_h \frac{P_l^{2/\alpha}}{P_h^{2/\alpha}} - \eta'_s$. The upper bound λ_s^+ is expressed in (12). The solution of problem (27) is summarized in the following proposition.

Proposition 2. The optimal spatial density of LPU that maximizes (27), given P_h , P_l , and λ_h , is expressed as follows,

$$\hat{\lambda}_l = \min\{\lambda_l^o, \lambda_s^+\}, \quad (28)$$

where

$$\lambda_l^o = \left[\frac{1}{\eta_l'} + \frac{1}{d_s} W \left[-\frac{c_s d_s (\eta_l' + d_s)}{a_s \eta_l'} \exp \left(-\frac{d_s}{\eta_l'} \right) \right] \right]^+ \quad (29)$$

Proof. To proof this proposition it is enough to show that \mathcal{E}_{TS} is strictly quasi-concave with respect to λ_l and has a single stationary (critical) point (details of the proof are omitted). \square

We optimize HPU's transmission power in similar lines as in optimizing the LPU's parameters. We apply the change of variable to P_h such that $P_u = P_h^{\frac{2}{\alpha}}$. We then introduce a new optimization variable related to the HPU's transmission power, i.e., P_{us} , and the associated equality constraint $P_{us} = P_u$. The optimal value of P_u which maximizes problem (20) is derived in the following proposition.

Proposition 3. *The optimal value of P_u that maximizes (20), given P_l and λ_l , is expressed as follows,*

$$\hat{P}_u = \min\{\max\{P_u^o, (P_p^-)^{\frac{2}{\alpha}}\}, (P_p^+)^{\frac{2}{\alpha}}\} \quad (30)$$

where

$$P_u^o = \left[\frac{\alpha + \sqrt{\alpha^2 - 16\mu_p \eta_p' \lambda_l P_l^{\frac{2}{\alpha}}}}{4\mu_h} \right]^{\frac{\alpha}{2}} \quad (31)$$

and P_p^+ and P_p^- are found in similar lines with (7) and (8), respectively. Whereas, the optimal value of P_{us} that maximizes (20), given P_l , λ_l , and λ_h , is expressed as follows,

$$\hat{P}_{us} = \min\{\max\{P_{us}^o, (P_s^-)^{\frac{2}{\alpha}}\}, (P_s^+)^{\frac{2}{\alpha}}\} \quad (32)$$

where

$$P_{us}^o = \frac{P_l^{\frac{2}{\alpha}}}{\eta_s' \lambda_h} \log \left[\frac{\eta_s' \lambda_h \lambda_l \alpha_s e^{-\eta_s' \lambda_l}}{\mu_p k_t P_l^{\frac{2}{\alpha} + 1}} \right] \quad (33)$$

The parameter μ_p is the Lagrangian multiplier associated with the equality constraint similar to that introduced for the LPU's power optimization case, in (21b). The parameter μ_h is obtained by finding the zeros of the following function.

$$g(\mu_p) = \left[2w_p \mu_p - \frac{\mu_p P_l^{\frac{2}{\alpha}}}{\eta_l' \lambda_h} \log(\mu_p) - \frac{\alpha}{2} \right]^2 - \frac{\alpha^2}{4} + 4\mu_p \eta_h' \lambda_l P_l^{\frac{2}{\alpha}} \quad (34)$$

where $w_p = \frac{P_l^{\frac{2}{\alpha}}}{\eta_s' \lambda_h} \left[\log \left(\eta_s' \lambda_h \alpha_l \lambda_l e^{-\eta_s' \lambda_l} \right) - \log \left(k_t P_l^{1 + \frac{2}{\alpha}} \right) \right]$.

Proof. The proof is obtained using similar steps to the proof of Proposition 1. \square

In the following proposition, we find the value of λ_h which maximizes problem (20).

Proposition 4. *The optimal spatial density of HPU's system that maximizes (20), given P_h , P_l , and λ_l , is expressed as follows,*

$$\hat{\lambda}_h = \min\{\lambda_h^o, \lambda_p^+\} \quad (35)$$

The parameter λ_p^+ is found in similar lines with that in (12).

The expression of λ_l^o is obtained as follows,

$$\lambda_h^o = \left[\frac{1}{\eta_h'} + \frac{1}{d_p} W \left[-\frac{c_p d_p (\eta_h' + d_p)}{a_p \eta_h'} \exp \left(-\frac{d_p}{\eta_h'} \right) \right] \right]^+ \quad (36)$$

where $a_p = \frac{\alpha_h}{k_t P_h} \exp \left(-\eta_h' \lambda_l \frac{P_l^{\frac{2}{\alpha}}}{P_h^{\frac{2}{\alpha}}} \right)$, $c_p = \frac{\alpha_l}{k_t P_l} \exp(-\eta_l' \lambda_l)$, and $d_p = \eta_l' \frac{P_h^{\frac{2}{\alpha}}}{P_l^{\frac{2}{\alpha}}} - \eta_p'$.

Proof. It is straight forward to obtain the proof following similar steps as in the proof of Proposition 2. \square

After finding the optimal values of $\{P_n, P_{np}, \lambda_l, P_u, P_{us}, \lambda_h\}$ which, individually, maximizes (20), we introduce an algorithm that enables us to jointly maximize (20). This algorithm utilizes the individual structure of the main problem in (20) w.r.t. each optimization variables, i.e., no need to verify the joint structure. The proposed algorithm, in Algorithm 1, iterates over the variables and update them in each iteration to obtain a joint optimal set. The following theorem introduces the global optimal solution of problem (20).

Theorem 2. *Given the strict quasi-concave structure of problem (20) w.r.t. each of the optimization variable, the global optimal solution of problem (20) is found through two steps. First, find the optimal variables $(P_l, \lambda_l, P_h, \lambda_h)$ as in (22), (28), (30), (35), respectively. Second, update and iterate over these variables $(P_l, \lambda_l, P_h, \lambda_h)$ using the alternating algorithm, in Algorithm 1.*

Proof. The proof is given in Appendix B. \square

Algorithm 1: Maximizing linearly combined \mathcal{E}_l and \mathcal{E}_h

input : $\eta_l', \eta_h', \epsilon_l, \epsilon_h, \alpha, \alpha_s, \alpha_p, k_t, P_{pk}, \epsilon$
1 Initialize: $\lambda_l^{(0)} = \frac{1}{\eta_s'}, \lambda_h^{(0)} = \frac{1}{\eta_h'}, P_l^{(0)} = P_{pk}, P_h^{(0)} = P_{pk}$,
 $cond = True, q = 1$
2 while $cond$ **do**
3 To find LPU's transmission power, we first solve for μ using (26), given fixed $P_h = P_h^{(q-1)}$, $\lambda_l = \lambda_l^{(q-1)}$, and $\lambda_h = \lambda_h^{(q-1)}$. By finding μ we guarantee that $P_n = P_{np}$. Thus, LPU's power is found as $P_l^{(q)} = \hat{P}_n$, in (22).
4 For the values of $P_l = P_l^{(q)}$, $P_h = P_h^{(q-1)}$, and $\lambda_h = \lambda_h^{(q-1)}$, find the LPU's spatial density, i.e., $\lambda_l^{(q)} = \hat{\lambda}_l$, as in (28).
5 Given $P_l = P_l^{(q)}$, $\lambda_l = \lambda_l^{(q)}$, and $\lambda_h = \lambda_h^{(q-1)}$, we find the parameter μ_p which guarantee that $P_u = P_{us}$. We then find $P_h^{(q)} = \hat{P}_u$ and $\lambda_h^{(q)} = \hat{\lambda}_h$ using (30) and (35).
6 Evaluate $\mathcal{E}_{TS}^{(q)} = \alpha_l \mathcal{E}_l(P_l^{(q)}, \lambda_l^{(q)}, P_h^{(q)}, \lambda_h^{(q)}) + \alpha_h \mathcal{E}_h(P_l^{(q)}, \lambda_l^{(q)}, P_h^{(q)}, \lambda_h^{(q)})$,
7 **if** $\left\| \mathcal{E}_{TS}^{(q)} - \mathcal{E}_{TS}^{(q-1)} \right\| < \epsilon$ **then:** $cond = False$
8 $q = q + 1$;
9 end
output: $\{P_l^{(q)}, \lambda_l^{(q)}, P_h^{(q)}, \lambda_h^{(q)}\}$

Note that P_{pk} , mentioned at the initialization stage of the algorithm is the maximum allowable transmission power.

V. NUMERICAL EVALUATION

In this section, we evaluate the EE performance of the proposed problems, i.e., \mathcal{E}_l (Sec. III) and \mathcal{E}_{TS} (Sec. IV). Note that through out the numerical results we use the following notations. The legend OPL is used to note that this result is associated with optimizing both spatial density and transmission power of the targeted system, whereas, OP or OL are used to note that this result is obtained by optimizing only the transmission power or the spatial density of the targeted system, respectively. The performance measure EE_l represents the LPU's EE as formulated in Sec. III, whereas, EE_{TS} represents the linear combination of both LPU and HPU systems' EE as formulated in Sec. IV. It is worth noting that under EE_{TS} scenario, the OPL and OP schemes optimize (in addition to spatial densities) w.r.t. $\zeta = \frac{P_l}{P_h}$ not P_l and P_h , individually. Hence, the denominator of \mathcal{E}_l and \mathcal{E}_h in (20), becomes $P_l = \zeta P_h$ and P_h , respectively, since we assume that the HPS's power is not likely to be changed. The reason behind this substitution is that we find it more numerically stable to optimize w.r.t. to ζ . The unit of EE is successful transmission/watt/unit-area, it could be converted to nats/joule/unit-area by multiplying the objective by $\log(1+\theta_s)$.

Figure 1(a) evaluates EE, EE_l , against tolerance parameters ϵ_l and ϵ_h , for different schemes (i.e., OPL, OL, and OP) and different parameter sets of $\{\epsilon_l, \epsilon_h\}$, i.e., $\{v, 0.1\}$, $\{v, 0.9\}$, $\{0.9, v\}$, ('v' is to note variable ϵ_l or ϵ_h). In general, it is observed that EE_l improves with the increase of ϵ_l and ϵ_h (since the feasibility region becomes larger), except for OP with $\epsilon_l = 0.9$ where both constraints are always inactive. Also, as expected OPL outperforms both OL and OP schemes. Under OL scheme, we note an increase in EE_l up-to a certain point, (a) or (b), then the performance saturates. This is because the constraint related to ϵ_h is active at (a), whereas, the one related to ϵ_l becomes inactive and the optimal λ_l has been achieved at (b). Point (a) occurred before (b) because of the difference in constraint tolerance, $\epsilon_h = 0.1, 0.9$, respectively. Same behavior is noted for OP scheme, as lower power constraint is inactive after point (c), where it was active before this point. As for point (d) on scheme OPL, the node density constraint related to ϵ_h is active for curve $\{v, 0.1\}$ (hence the optimal λ_l is constant). Yet, improvement could be made for both ($\{v, 0.1\}$ & $\{v, 0.9\}$) curves because of the impact of increasing ϵ_l on power lower bound, P_s^- . The saturation after point (e) on both curves ($\{v, 0.1\}$ & $\{v, 0.9\}$) occurs because there is no impact of ϵ_l on the optimal power. Similar behavior is inferred at (f).

Figure 1(b) evaluates the EE_l versus θ_l or θ_h , noted in the legend as $\{\theta_h, \theta_l\}$ and set as $\{v, 10.5\}$, $\{v, 0.5\}$, $\{10.5, v\}$, $\{0.5, v\}$ ('v' is to note variable θ_l or θ_h). It is clear that EE_l is decreasing with θ , while variable θ_l has higher effect on the variance of EE_l , in compared to variable θ_h . Note that $\{v, 10.5\}$ intersects with $\{10.5, v\}$ around $\theta_l = 10.5$ for OPL, OL, and OP curves. Hence, to have relatively high EE_l (in compared to EE_h) it is preferred to set $\theta_l < \theta_h$. As expected, it is observed that OPL scheme outperforms both OP and OL for all values of θ_l and θ_h .

Figure 1(c) evaluates EE, EE_{TS} , against variable ϵ_l and ϵ_h , for both OL and OP schemes, under several variation of power and weight sets, i.e., $\{P_l, P_h, \alpha_h\} = \{15, 5, 0.5\}, \{5, 15, 0.5\}, \{15, 15, 0.5\}, \{0, 15, 0.5\}, \{0, 15, 0.9\}$. Note that Unlike Fig. 1(a), both ϵ_l and ϵ_h change similarly, as an x-axis. A general observation for both schemes is that if the P_l/P_h ratio is high or low ($\gg 1$ or $\ll 1$), then the dominating EE is the one corresponding to the lower power (if P_l/P_h is low it follows that EE_l is dominating). However, if P_l/P_h is close to one, then the dominating EE is the one with higher weight, α_l or α_h . Also, the OP performance for different curves of $\{5, 15, 0.5\}, \{15, 15, 0.5\}, \{0, 15, 0.5\}$ is the same because we model the denominator of \mathcal{E}_h and \mathcal{E}_l to be $k_t P_h$ and $k_t P_l = k_t \zeta P_h$, respectively, (fixing P_h while optimizing the ratio ζ). Hence, the critical parameters are the constant P_h and α_h . We observe that for OL curves decreasing the ratio P_l/P_h will shift the intersection point of the curves OL and OP to the right, i.e., it requires higher tolerance, ϵ_l and ϵ_h , for OP to outperform OL.

VI. CONCLUSION

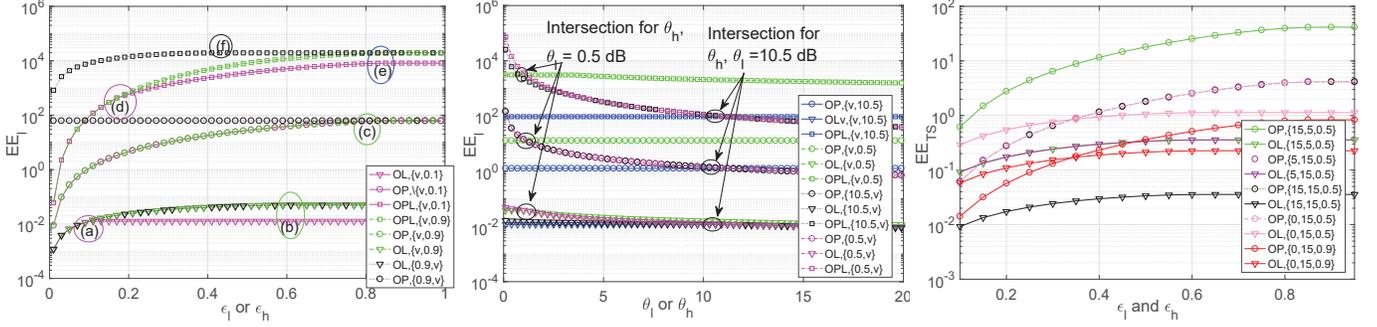
In this work, we maximize the energy efficiency of a DD spectrum sharing, in IoT environment, where multiple LPUs and HPU's co-exist in the same area. We construct a multi-objective optimization problem to maximize the combined HP's and LP's EE performance. Analytical expressions are provided for the optimal variables under many scenarios. Numerical results have shown a large gain by jointly optimizing the spatial density and transmission power in compared to individual optimization, reaches up-to 2-4 order of magnitude. It also showed that the dominant system which decides the overall EE is the system corresponding to the lowest power for low/high power ratio.

APPENDIX A PROOF OF PROPOSITION 1

To proof Proposition 1, we must show that utilizing the generalized theory of convex optimization, i.e., Lagrangian function and KKT conditions, applies to problem (21). Hence, in here, we proof the strict quasi-concavity of \mathcal{E}_{TS} in P_n and P_{np} . The necessity of proving the strictness is because in Alg. 1 we will utilize this property to propose an iterative global solution for problem 20. The proof of strict quasi-concavity of \mathcal{E}_{TS} with respect to P_n is easily derived, many work have done close proofs [4]. Whereas, the strict quasi-concavity of \mathcal{E}_{TS} with respect to P_{np} is easily observed since the function is strictly decreasing on P_{np} . To derive the expression in (23), we must observe that maximizing \mathcal{E}_{TS} with respect to P_n is equivalent to the following problem,

$$\max_{(P_s^-)^{\frac{2}{\alpha}} \leq P_n \leq (P_s^+)^{\frac{2}{\alpha}}} \alpha_l \lambda_l e^{\left(-\eta_l \left(\lambda_l + \lambda_h \frac{P_h \alpha}{P_n}\right)\right)} \Bigg/ \left(k_t P_n^{\frac{\alpha}{2}}\right) \quad (37a)$$

$$\text{s.t.} \quad P_n = P_{np} \quad (37b)$$



(a) LPS's EE versus tolerance threshold. Param- (b) LPS's EE versus SIR. Parameters: $P_{l,h} = 25\text{dBm}, \theta_{l,h}=10.5\text{dB}, R_{l,h}=2\text{m}, k_t=1, \lambda_{l,h}=0.5\text{e-}4$. (c) Combined LPS and HPS EE versus tolerance threshold. Parameters: $R_{l,h}=4\text{m}, \epsilon_{l,h} = 0.2, \theta_{l,h}=4\text{dB}, k_t=1, \lambda_{l,h}=1\text{e-}4$.

Fig. 1. Overall Energy Efficiency Numerical Results

Note that the solution of this problem is different than that of (9) because we including the equality constraint $P_n = P_{np}$. We then use the transformation on the objective function in (37a) using monotone increasing function, i.e., $\log(\cdot)$. The corresponding Lagrangian function of problem (37) is expressed as follows,

$$\mathcal{L} = \log\left(\frac{\alpha_s \lambda_l}{k_t}\right) - \eta'_s \lambda_l - \eta'_s \lambda_h \frac{P_u}{P_n} - \frac{\alpha}{2} \log(P_n) + \mu [P_n - P_{np}], \quad (38)$$

where $P_u = P_h^{\frac{2}{\alpha}}$. Taking the derivative of the Lagrangian function in (38) and finding its zeros results in the following,

$$2\mu P_n^2 - \alpha P_n + 2\eta'_s \lambda_h P_u = 0. \quad (39)$$

Hence, the expression in (23) is obtained by solving (39).

We then derive the expression in (25), by observing that maximizing \mathcal{E}_{TS} with respect to P_{np} is equivalent to the following problem,

$$\max_{(P_s^-)^{\frac{2}{\alpha}} \leq P_{np} \leq (P_s^+)^{\frac{2}{\alpha}}} \alpha_h \lambda_h e^{\left(-\eta'_h \left(\lambda_h + \frac{\lambda_l P_{np}}{P_h^{\frac{\alpha}{2}}}\right)\right)} / (k_t P_h) \quad (40a)$$

$$\text{s.t.} \quad P_n = P_{np}. \quad (40b)$$

We then formulate the Lagrangian function of (40), devise its first derivative and find its zeros as follows,

$$\frac{-\alpha_p \lambda_h \eta'_p \lambda_l e^{-\eta'_p \lambda_h}}{k_t P_u^{\frac{\alpha}{2}}} \exp\left(\frac{-\eta'_p \lambda_l P_{np}}{P_u}\right) + \mu = 0. \quad (41)$$

Solving (41) leads to the exact solution in (25).

APPENDIX B PROOF OF THEOREM 2

To proof Theorem we need to satisfies the conditions of the findings in [14]. Grippo and Sciandrone, in [14], verified that using Gauss-Seidel method to optimize over several variable guarantee a global optimal solution without the necessity of proving joint concavity/convexity structure. The condition which must be met is to prove that the targeted problem is strict quasi-concave with respect to to each individual variable. Therefore, to proof B, it is enough to verify the strict quasi-concave property of problem \mathcal{E}_{TS} with respect to each variable

$P_n, P_{np}, P_u, P_{us}, \lambda_l, \lambda_{sp}$. However, note that we have already proven the strict quasi-concave of the problem with respect to P_n, P_{np} , and λ_l in the associated appendices. Similarly, we can prove the strict quasi-concave structure of \mathcal{E}_{TS} with respect to P_u, P_{us} , and λ_h . This concludes the proof of Theorem 2.

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