Energy Efficient Cross Layer Design for Spectrum Sharing Systems

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***Abstract*—We propose a cross layer design that optimizes the energy efficiency of spectrum sharing systems. The energy per good bit (EPG) is considered as an energy efficiency metric. We optimize the secondary user's transmission power and media access frame length to minimize the EPG metric. We protect the primary user transmission via an outage probability constraint. The non-convex targeted problem is optimized by utilizing the generalized convexity theory and verifying the strictly pseudo-convex structure of the problem. Analytical results of the optimal power and frame length are derived. We also used these results in proposing an algorithm, which guarantees the existence of a global optimal solution. Selected numerical results show the improvement of the proposed system compared to other systems.**

***Index Terms*—Spectrum sharing, Energy efficiency, Resource allocation, Cross layer design,**

I. Introduction

The energy consumption of information and communication technology (ICT) sector has been dramatically increasing. Therefore, researchers have been thoroughly investigating the greenness of the ICT. In order to improve the communication system's greenness, several energy efficiency (EE) metrics have been investigated and analyzed; yet, there is not a formal framework which specifies the best notion of greenness. Researchers have addressed several EE metrics through different factors based on different communication layers, e.g., routing, physical (PHY), media access control (MAC) layers, etc.

In MAC layer, several work have been conducted to improve energy efficiency. In [1], authors assumed a quasi-static Rayleigh fading channel (i.e., channel does not change during re-transmission, but it may change independently for new transmissions). Considering a hybrid ARQ (H-ARQ) protocol, authors of [1] devised an optimal power allocation that is obtained recursively for each of the retransmission rounds. Authors of [2] proposed S-MAC to reduce energy consumption in wireless sensor networks. S-MAC enables nodes to operate on a low-duty-cycle, introducing sleeping time to reserve energy. It also re-introduces the message-passing concept to save energy through controlling the overhead. Authors of [3] addressed the energy efficiency metric from a cross-layer design perspective. They minimized the transmission energy and normalized transmission energy, where the normalization parameter is a function of retransmission factor, frame length, modulation index and channel coding.

On the other hand, researchers have addressed the PHY layer issues of energy efficiency. In [4], authors studied the tradeoff between the energy efficiency and the spectral efficiency. This study addressed the fairness issue among different orthogonal frequency-division multiple access (OFDMA) users. The corresponding near-optimal power allocation of the targeted tradeoff has been derived. In [5], authors tackled the issue of EE (capacity to power ratio) in a multiple-input and multiple-output (MIMO) channel under cognitive radio (CR) settings. They converted the problem into a semi-definite programming (SDP) problem and obtained the associated optimal beamforming vector under different assumptions on the availability of the channel state information (CSI). Also, under MIMO environment, authors of [6] formulated the EE problem as a minimization of power constrained by minimum rate constraint. They derived the optimal beamforming vector and the associated time slot in a time division multiple access (TDMA) network.

In this work, we formulate the problem under a CR environment. The primary user (PU) is protected by enforcing it outage probability to be lower than a certain threshold. We showed that the targeted energy per good bit of secondary user (SU) is not a convex function. However, we verified that this problem is strictly pseudo-convex and strictly quasi-convex with respect to each variable. Using this structure, analytical expressions of the optimal power and frame length allocation have been devised. Utilizing these expressions we propose an algorithm that guarantee a global optimal solution to the problem. Unlike the work in [3], we consider energy per goodbit (EPG) metric, i.e., average power divided by the Shannon capacity metric as the targeted energy efficiency metric. In addition, we consider the retransmission parameter, which depends on the modulation index, channel coding scheme, packet length, and overhead length.

II. System Model

A. System Model

We consider a CR system where all terminals have a single antenna, i.e., single-input single-output (SISO) system. One pair of secondary users, i.e., secondary transmitter (ST) and secondary receiver (SR), and one pair of primary users, i.e., primary transmitter (PT) and primary receiver (PR), are considered in this model. The SU's and PU's channels are designated as h_s and h_p, whereas the interference channel links from ST to PR and PT to SR are designated as h_{sp} and

h_{ps}. The squared modulus of the channels are expressed as $\gamma_{ps} = |h_{ps}|^2$, $\gamma_{sp} = |h_{sp}|^2$, $\gamma_s = |h_s|^2$ and $\gamma_p = |h_p|^2$. The corresponding means of the channels gains are μ_{ps}, μ_{sp}, μ_s and μ_p, respectively, whereas, the variances are expressed as σ^2_{ps}, σ^2_{sp}, σ^2_s and σ^2_p, respectively. The previously mentioned channel's gains are assumed to be independent. Through a feedback channel, we assume that both ST and SR share instantaneous CSI about the SU channel γ_s. It is assumed that PT transmits with a maximum power policy P_p. This is to consider the worst case scenario of PT to SR interference. ST has an adaptive power allocation policy (to be described later) denoted as P_s. SU decodes the PU interference as noise, since it does not know γ_{ps}, hence, it is unable to perform any interference mitigation technique. Considering that our system operates in a CR environment, we have to protect the PU from the SU's interference. Therefore, we enforce an outage probability interference constraint on the SU's transmission. The proposed system considers a sophisticated framework, which takes into account optimizing two variables, i.e., SU's transmission power and SU's MAC layer's frame length. Hence, it increases the problem's degrees of freedom.

III. Problem Formulation

In this section, we present the problem formulation, which introduces the main framework of the proposed scheme. We begin by introducing the targeted optimization problem and the necessary assumptions to solve it. We then study the problem structure. We verify that the problem in its original form is not convex with respect to the optimization variables. The strict pseudo-convexity structure of the problem is then verified.

The proposed EPG minimization problem is expressed as,

$$\mathfrak{P}_0 : \min_{P_s, L} \mathcal{E}_e\left(P_s, L\right) = \frac{\mathfrak{R}_t\left[\frac{L_F}{L} P_t P_s + P_c\right]}{\frac{L}{L_F}\log(1+\frac{P_s\gamma_s}{1+P_p\sigma^2_{ps}})} \tag{1a}$$

$$\text{s.t.} \quad \mathfrak{C}_1: \Pr\left[\mathcal{I}_p < R_p \middle| \gamma_s\right] \leq \epsilon_p. \tag{1b}$$

$$\mathfrak{C}_2: P_s \leq P_{pk}. \tag{1c}$$

where P_c and P_t are assigned parameters which are associated with the circuit power of the radio devices and power amplifier constant power consumption. The MAC layer frame length $L_F = L + L_0$ includes the information bits, i.e., L, and the overhead bits, i.e., L_0. The transmission powers P_s and P_p are the associated power of SU and PU, respectively. Since we assume that SU does not have knowledge of γ_{ps}, the mutual information of SU is derived along similar lines as in [7], i.e., $\log(1+\frac{P_s\gamma_s}{1+P_p\sigma^2_{ps}})$ [1]. Let us note $P_I = 1 + P_p\sigma^2_{ps}$. Similar to SU case, the PU's mutual information is expressed as $\mathcal{I}_p = \log\left(1+\frac{P_p\gamma_p}{1+P_s\sigma^2_{sp}}\right)$. $\mathfrak{R}_t$ is the average number of retransmissions of the frame, which is expressed as, $\mathfrak{R}_t = \frac{1}{1-\mathfrak{F}_{er}}$, where $\mathfrak{F}_{er}$ is the frame error rate, approximated under a Rayleigh fading channel as [8],

$$\mathfrak{F}_{er} \simeq 1 - \exp\left(-\frac{\gamma_w}{\gamma_b}\right), \tag{2}$$

[1]The parameter $1 + P_p\sigma^2_{ps}$ can be easily obtained via sensing PU's signal, i.e., $\mathbb{E}\{y_{ps}y^*_{ps}\} = 1 + P_p\sigma^2_{ps}$, where y_{ps} is PU's signal at SU sensor.

where $\gamma_b = \frac{P_s}{1+P_p\sigma^2_{ps}}$. The threshold γ_w is approximated as,

$$\gamma_w \simeq k_M \log\left(L_F\right) + b_M. \tag{3}$$

where k_M and b_M are the related parameters to the coding and modulation schemes [3]. Finally, the number of retransmissions is rewritten as,

$$\mathfrak{R}_t \simeq \exp\left(\frac{k_M\log\left(L_F\right)+b_M}{\gamma_b}\right). \tag{4}$$

It is clear that both constraints $\mathfrak{C}_1$ and $\mathfrak{C}_2$ can be combined, thus $\mathfrak{P}_0$ is rewritten as follows,

$$\mathfrak{P}_0 : \min_{P_s, L} \mathcal{E}_e\left(P_s, L\right) = \frac{\mathfrak{R}_t\frac{L_F}{L}\left[\frac{L_F}{L} P_t P_s + P_c\right]}{\log(1+\frac{P_s\gamma_s}{P_I})} \tag{5a}$$

$$\text{s.t.} \qquad P_s \leq \min\left\{Q_I, P_{pk}\right\} = P^{st}\ . \tag{5b}$$

Note that the capacity normalization parameter $\frac{L}{L_F}$ is moved to the numerator. Constraints $\mathfrak{C}_1$ and $\mathfrak{C}_2$ are converted into a short-term power constraint, i.e., (5b). The short-term power constraint Q_I is obtained from $\mathfrak{C}_1$ as follows,

$$\Pr\left[\frac{1}{2}\log\left(1+\frac{P_p\gamma_p}{P_s(\gamma_s)\sigma^2_{sp}+1}\right) < R_p \middle| \gamma_s\right] \leq \epsilon \tag{6a}$$

$$\implies F_{\gamma_p|\gamma_s}\left(\gamma_p \leq \frac{\left(P_s(\gamma_s)\sigma^2_{sp}+1\right)\left(e^{R_p}-1\right)}{P_p}\right) \leq \epsilon \tag{6b}$$

$$\implies P_s\left(\gamma_s\right) \leq \left[\frac{F^{-1}_{\gamma_p}\left(\epsilon\right)P_p}{(e^{R_p}-1)\sigma^2_{sp}} - \frac{1}{\sigma^2_{sp}}\right]^+ = Q_I, \tag{6c}$$

where (6c) is obtained from the independence between γ_p and γ_s and from the fact that $F_{\gamma_p|\gamma_s}$, being a cumulative density function (CDF), is a monotonically non-decreasing function. Note that PU knows the channel gain γ_p, however, this constraint is enforced at SU's side. Therefore, we condition on γ_s, while considering γ_p as the random variable.

In order to analyze the objective function, EPG, we begin by verifying that it is not convex for all values of P_s and L. Rewriting the numerator of $\mathcal{E}_e(P_s, L)$ as follows,

$$\begin{aligned}\mathcal{E}_n\left(P_s, L\right) &= e^{\left(\frac{a(L)}{P_s}\right)}\frac{L_F}{L}\left[\frac{L_F}{L}P_tP_s + P_c\right] \\ &= \mathcal{E}^{(1)}_n\left(P_s, L\right) + \mathcal{E}^{(2)}_n\left(P_s, L\right),\end{aligned} \tag{7}$$

where $a(L) = k'_M\log\left(L_F\right) + b'_M$, $k'_M = k_M P_I$ and $b'_M = b_M P_I$. The terms $\mathcal{E}^{(1)}_n$ and $\mathcal{E}^{(2)}_n$ are defined as $\mathcal{E}^{(1)}_n\left(P_s, L\right) = \exp\left(\frac{a(L)}{P_s}\right)\left(\frac{L_F}{L}\right)^2 P_tP_s$ and $\mathcal{E}^{(2)}_n\left(P_s, L\right) = \exp\left(\frac{a(L)}{P_s}\right)\frac{L_F}{L}P_c$. Note that, hereafter, we use the notations $\mathcal{E}_n\left(P_s, L\right)$, $\mathcal{E}^{(1)}_n\left(P_s, L\right)$ and $\mathcal{E}^{(2)}_n\left(P_s, L\right)$ interchangeably with $\mathcal{E}_n$, $\mathcal{E}^{(1)}_n$ and $\mathcal{E}^{(2)}_n$. To check the convexity of EPG (with respect to L), we test the convexity of $\mathcal{E}_n$. Rigorously analyzing the structure of $\mathcal{E}^{(2)}_n$, we conclude that the convexity of $\mathcal{E}^{(2)}_n$ with respect to L depends on the value of the allocated power (unlike the results in [3]), as follows,

$$\forall \frac{k'_M}{P_s} > 1 \implies \begin{array}{ll}\mathcal{E}^{(2)}_n \text{ is convex if:} & \frac{L_0P_s}{k'_M - P_s} > L \\ \mathcal{E}^{(2)}_n \text{ is concave if:} & \frac{L_0P_s}{k'_M - P_s} < L\end{array} \tag{8a}$$

$$\forall \frac{k_M^{'}}{P_s} < 1 \implies \begin{array}{ll} \mathcal{E}_n^{(2)} \text{ is convex if: } & \frac{L_0 P_s}{k_M^{'} - P_s} < L \\ \mathcal{E}_n^{(2)} \text{ is concave if: } & \frac{L_0 P_s}{k_M^{'} - P_s} > L. \end{array} \tag{8b}$$

The results in (8) verifies that $\mathcal{E}_n^{(2)}$ is not a convex function in L. Following a similar line it is easy to show that $\mathcal{E}_n^{(1)}$ is also not convex in L. It is also noted that $\mathcal{E}_e$ is a fractional non-convex function of P_s. Therefore, it is necessary to find an alternative structure of $\mathcal{E}_e$, with respect to P_s and L, to guarantee a global optimal solution. In [9], it is proven that to utilize the Lagrangian method in finding the global optimal variables it is sufficient to prove the pseudo-convexity structure of the problem with respect to its variables and satisfy the Karush-Kuhn-Tucker (KKT) conditions. Thus, we verify the strict pseudo-convexity structure of $\mathcal{E}_e$ with respect to each of the optimization variables, i.e., P_s and L, respectively[2].

Lemma 1. *The numerator of problem $\mathfrak{P}_0$ is strictly convex with respect to P_s.*

Proof: It is clear that $\mathcal{E}_n^{(2)}$ is a strictly convex function with respect to P_s. To prove the strict convexity property of $\mathcal{E}_n^{(1)}$ we derive its first order derivative as follows,

$$\frac{\partial \mathcal{E}_n}{\partial P_s} = e^{\left(\frac{a(L)}{P_s}\right)} \frac{L_F}{L} P_t - a(L) e^{\left(\frac{a(L)}{P_s}\right)} \frac{L_F}{L P_s} P_t. \tag{9}$$

We then devise the second order derivative as follows,

$$\frac{\partial^2 \mathcal{E}_n}{\partial P_s^2} = e^{\left(\frac{a(L)}{P_s}\right)} \frac{P_t L_F}{L P_s^3} \left[-P_s + a^2(L) + a(L) P_s\right] > 0. \tag{10}$$

It is enough to recall that $a(L) > 1$ to verify (10). Since the sum of two strictly convex function, $\mathcal{E}_n^{(1)}$ and $\mathcal{E}_n^{(2)}$, results in strictly convex function. Then, it is clear that $\mathcal{E}_n$ is strictly convex function with respect to P_s. ■

It is observed that $\log(1 + \frac{P_s \gamma_s}{P_I})$ is strictly concave with respect to P_s. In the following proposition, to verify the strictly pseudo-convex structure of $\mathcal{E}_e$, we utilize the fact that the numerator and denominator of $\mathcal{E}_e$ are strictly convex and strictly concave with respect to P_s, respectively.

Proposition 1. *The objective function of problem $\mathfrak{P}_0$, $\mathcal{E}_e(P_s)$, is strictly pseudo-convex with respect to P_s.*

Proof: In order to prove proposition 1, we show that the ratio between a strictly convex function and a strictly concave function results in a strictly pseudo-convex function. To show that $\mathcal{E}_e(P_s)$ is strictly pseudo-convex, two conditions must be satisfied [9],

1) $\mathcal{E}_e(P_s)$ is a strictly quasi-convex function.
2) There exist a local minimum P_s^*, i.e., $\nabla \mathcal{E}_e(P_s) = 0$.

A strictly quasi-convex function is defined as follows, $\mathcal{E}_e\left(\lambda P_s^{(1)} + (1-\lambda) P_s^{(2)}\right) < \max\left\{\mathcal{E}_e(P_s^{(1)}), \mathcal{E}_e(P_s^{(2)})\right\}$. For the sake of analysis, we rewrite $\mathcal{E}_e(\mathbf{P_s})$ as a ratio between a numerator and a denominator terms as follows, $\mathcal{E}_e(P_s) = \frac{\mathcal{E}_n(P_s)}{\mathcal{E}_d(P_s)} = \frac{\mathfrak{R}_t \frac{L_F}{L}\left[\frac{L_F}{L} P_t P_s + P_c\right]}{\log(1+\frac{P_s \gamma_s}{P_I})}$. Strict convexity and concavity properties of each $\mathcal{E}_n(P_s)$ and $\mathcal{E}_d(P_s)$ are utilized to show the strict quasi-convexity of $\mathcal{E}_e(P_s)$ as follows,

$$\mathcal{E}_n\left(\lambda P_s^{(1)} + (1-\lambda) P_s^{(2)}\right) \tag{11a}$$

$$< \lambda \mathcal{E}_n\left(P_s^{(1)}\right) + (1-\lambda) \mathcal{E}_n\left(P_s^{(2)}\right) \tag{11b}$$

$$< \lambda \mathcal{E}_n\left(P_s^{(1)}\right) + (1-\lambda) \frac{\mathcal{E}_n\left(P_s^{(1)}\right)}{\mathcal{E}_d\left(P_s^{(1)}\right)} \mathcal{E}_d\left(P_s^{(2)}\right) \tag{11c}$$

$$< \frac{\mathcal{E}_n\left(P_s^{(1)}\right)}{\mathcal{E}_d\left(P_s^{(1)}\right)} \left[\mathcal{E}_d\left(\lambda P_s^{(1)} + (1-\lambda) P_s^{(2)}\right)\right] \tag{11d}$$

$$\implies \mathcal{E}_e\left(\lambda P_s^{(1)} + (1-\lambda) P_s^{(2)}\right) \leq \mathcal{E}_e\left(P_s^{(1)}\right), \tag{11e}$$

where (11c) results by assuming that $\mathcal{E}_e(P_s^{(2)}) < \mathcal{E}_e\left(P_s^{(1)}\right)$, (11d) is valid because of the strict concave property of $\mathcal{E}_d(P_s)$.

The second step in the proof is to show that $\nabla \mathcal{E}_e(P_s^*) = 0$. This step is rigorously derived in Appendix A [11].

By combining (11) and the results from Appendix A, we conclude that the function $\mathcal{E}_e$ is strictly pseudo-convex. Hence, utilizing the results from [9] the P_s minimizer of problem $\mathfrak{P}_0$ is obtained by satisfying the KKT conditions of the corresponding problem.

■

Proposition 2. *The objective function of problem $\mathfrak{P}_0$ is strictly pseudo-convex with respect to L.*

Proof: The strict quasi-convexity structure of $\mathcal{E}_e(L) = \mathcal{E}_n^{(1)} + \mathcal{E}_n^{(2)}$ is verified by showing that $\mathcal{E}_e(L)$ is a decreasing function or increasing function or decreasing then increasing function of L [9]. The first derivative of $\mathcal{E}_e(L)$ is expressed as follows,

$$\frac{\partial \mathcal{E}_e(L)}{\partial L} = \frac{k_M^{'}}{P_s} e^{\frac{k_M^{'} \log(L_F) + b_M^{'}}{P_s}} \frac{L_F}{L^2} + e^{\frac{k_M^{'} \log(L_F) + b_M^{'}}{P_s}} \frac{L_F}{L}. \tag{12}$$

Let us note $\mathcal{E}_e \uparrow$ for an increasing $\mathcal{E}_e$ and $\mathcal{E}_e \downarrow$ for a decreasing $\mathcal{E}_e$. The behavior of $\mathcal{E}_e(L)$ is studied through the following inequality,

$$\frac{k_M^{'}}{P_s} L_F L - 2 L_F L_0 + \frac{k_M^{'}}{P_s} L^2 - L_0 L \underset{\mathcal{E}_e\downarrow}{\overset{\mathcal{E}_e\uparrow}{\gtrless}} 0. \tag{13}$$

Note that as $L \to \infty$ the function $\mathcal{E}_e$ is increasing function,

$$\lim_{L\to\infty} \frac{\partial \mathcal{E}_e(L)}{\partial L} > 0. \tag{14}$$

However, as $L \to L_0$, then,

$$\begin{array}{ll} \frac{k_M^{'}}{P_s} > \frac{5}{3} & \implies \mathcal{E}_e(L) \text{ is increasing} \\ \frac{k_M^{'}}{P_s} < \frac{5}{3} & \implies \mathcal{E}_e(L) \text{ is decreasing} \end{array}. \tag{15}$$

[2]However, note that verifying the pseudo-convexity structure, separately, with respect to each variable does not result in a jointly global optimal solution. Therefore, we verify the strictness of the function. This strict pseudo-convex structure enables us to utilize the results in [10]. Hence, we propose an alternating algorithm to iterate over each optimal solution (optimal with respect to each variable) until it reaches a global joint optimal solution [10].

For general L, it is noted that,

$$\begin{array}{lll} L \in [0\ ,\ L_2^*] & \Longrightarrow \mathcal{E}_e(L) \text{ is decreasing} \\ L \in [L_2^*\ ,\ \infty] & \Longrightarrow \mathcal{E}_e(L) \text{ is increasing} \end{array}, \quad (16)$$

where $L_2^* = L_0 \left[3 - \frac{k_M'}{P_s} + \sqrt{\left(\frac{k_M'}{P_s}\right)^2 + 10\frac{k_M'}{P_s} + 9}\right]$. Therefore, combining (14), (15), and (16) it is clear that $\mathcal{E}_e(L)$ is a strictly decreasing function then strictly increasing function. Thus, $\mathcal{E}_e(L)$ is a strictly quasi-convex with respect to L. Since $\frac{\partial \mathcal{E}_e(L)}{\partial L} = 0$ at L_2^*, then, $\mathcal{E}_e(L)$ is also a pseudo-convex [9]. Since $\mathcal{E}_e(L)$ is both strictly quasi-convex and pseudo-convex, then $\mathcal{E}_e(L)$ is strictly pseudo-convex with respect to L.

■

IV. Methodology

In this section, we provide the optimal power policy and frame length that achieve minimum $\mathcal{E}_e(P_s, L)$. We then propose an alternating algorithm to guarantee a global optimal solution.

We begin by transforming problem $\mathfrak{P}_0$ to its epigraph form as follows, $\mathfrak{P}_0'$:

$$\min_{t, P_s, L}\ t \quad (17a)$$

$$\text{s.t.} \quad P_s \leq P^{st} \quad (17b)$$

$$\mathfrak{R}_t \frac{L_F}{L}\left[\frac{L_F}{L} P_t P_s + P_c\right] - t\log(1 + \frac{P_s \gamma_s}{P_I}) \leq 0. \quad (17c)$$

Due to the existence of $\frac{1}{P_s}$ in the exponential term (in $\mathfrak{R}_t$) and inside the logarithm in (17c), the expression of the optimal P_s is difficult to obtain. Therefore, we change $\mathfrak{P}_0'$ to an equivalent problem by introducing a new optimization variable, P_{bn}, and an equality constraint as follows, $\mathfrak{P}_1$:

$$\min_{t, P_s, P_{bn}, L}\ t \quad (18a)$$

$$\text{s.t.}: e^{\left(\frac{a(L)}{P_{bn}}\right)} \frac{L_F}{L}\left[\frac{L_F}{L} P_t P_{bn} + P_c\right] - t\log\left(1 + \frac{P_s \gamma_s}{P_I}\right) \leq 0 \quad (18b)$$

$$P_{bn} = P_s \quad (18c)$$

$$P_s \leq P^{st}. \quad (18d)$$

Introducing the equality constraint, in (18c), is the key enabler of our solution methodology. This is because it separates the P_s term in the exponential and the P_s term in the logarithm.

The corresponding Lagrangian function of problem $\mathfrak{P}_1$ is expressed as follows,

$$\mathfrak{L} = t - \lambda\left[e^{\frac{\gamma_w P_I}{P_{bn}}} \frac{L_F}{L}\left[\frac{L_F}{L} P_t P_{bn} + P_c\right] - t\log(1 + \frac{P_s \gamma_s}{P_I})\right] - \mu\left[P_{bn} - P_s\right]. \quad (19)$$

The optimal P_s is expressed as follows,

$$P_s^* = \left[\frac{\lambda t L e^{-\left(\frac{\gamma_w P_I}{P_{bn}}\right)}}{\mu L_F} - \frac{P_I}{\gamma_s}\right]_0^{P^{st}}. \quad (20)$$

The optimal P_{bn} is expressed as follows,

$$P_{bn}^* = \left[\frac{\gamma_w P_I}{2W\left[\frac{b\gamma_w P_I}{2a}\sqrt{\frac{a}{b}}\right]}\right]_0^{P^{st}}, \quad (21)$$

where $a = \frac{\lambda t L}{L_F}\log(1 + \frac{P_s \gamma_s}{1 + P_p \sigma_{ps}^2})$ and $b = \frac{L_F}{L} P_t \lambda - \mu$. The optimal value of t is obtained numerically using a bi-sectional algorithm. This guarantees an optimal solution because the problem is quasi-convex. The optimal frame length is expressed as follows,

$$L^* = \frac{L_0 \frac{P_s}{k_M P_I}}{2\left(P_c + P_{bn} P_t\right)}\Big[-\left(k_M P_I P_t - 2P_{bn} P_t - P_c\right) + \sqrt{\left(k_M P_I P_t - 2P_{bn} P_t - P_c\right)^2 + 8P_t k_M P_I\left(P_c + P_t P_{bn}\right)}\Big]. \quad (22)$$

The optimal value of the Lagrangian variable μ is obtained by equating (21) and (20). Thus, μ^* is the root of the following equation,

$$f_\mu\left(\mu^*\right) = \frac{\gamma_w P_I}{2W\left[\frac{b\gamma_w P_I}{2a}\sqrt{\frac{a}{b}}\right]} - \frac{\lambda t L e^{-\left(\frac{\gamma_w P_I}{P_{bn}}\right)}}{\mu L_F} + \frac{P_I}{\gamma_s} = 0. \quad (23)$$

The optimal value of the Lagrangian variable λ is obtain by solving the KKT condition associated with (18b).

Finally, in order to guarantee a global joint optimal solution of P_s, P_{bn}, and L we propose, in Algorithm 1, an alternating optimization algorithm. Because of the strictly quasi-convex structure of the problem with respect to each variable, it is verified that the proposed algorithm results in jointly global minimum value of $\mathcal{E}_e$ [10].

Algorithm 1: Proposed Algorithm

```
input : δ, ε, α, P_t, P_c, k_M, b_M, P_pk, γ_s, P_I, L_0
Initialize: P_s^(0) = P_pk, P_bn^(0) = P_pk, L^(0) = L_max,
  cond = True;
q = 1
while cond do
    Find a feasible value of t using a bisectional algorithm.
    In parallel, find λ by solving the KKT conditions
      associated with (18b) and find μ* from (23), given fixed
      P_s = P_s^(q-1), P_bn = P_bn^(q-1), and L = L^(q-1).
    By finding μ* we guarantee that P_s = P_bn. Thus, SU's
      power is found as P_s^(q) = P_s^*, in (20), given L = L^(q-1).
    By substituting λ and μ* in (21) we find P_bn^(q) = P_bn^*,
      given L = L^(q-1).
    The optimal value of frame length, L*, is found using (22)
      given P_s = P_s^(q), P_bn = P_bn^(q).
    Evaluate E_e^(q)(P_s^(q), L^(q)) as in (5a).
    if ||E_e^(q) - E_e^(q-1)|| < δ then: cond = False
    q = q+1;
end
output: {P_s^(q), L^(q)}
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The parameter δ, in Algorithm 1, is the stopping criteria of the algorithm and L_{max} is the maximum frame length. In

Algorithm 1 we initialize t using the bisectional algorithm, and initialize P_s, P_{bn}, and L by initial values. It follows that we find both μ and λ. We then find, in order, $P_s^{(q)}$, $P_{bn}^{(q)}$, and $L^{(q)}$. Then, keep iterating and updating the solution until the difference between the current $\mathcal{E}_e^{(q)}$ and the previous one, $\mathcal{E}_e^{(q-1)}$, is within a pre decided threshold.

V. Numerical Evaluation

In this section, we numerically evaluate SU's EPG. We show the effect of changing PU's interference parameters on the energy efficiency metric. As our formulation of the energy efficiency problem is unique, we could not find any work that optimize both the power and frame length using similar objective function. Hence, the benchmark to our work is derived in two methods, i.e., separately optimizing the problem with respect to the transmission power (as in [4]) or with respect to the frame length subject to the previous mentioned constraint. We then show the improvement gained by optimizing both parameters, transmission power and frame length, in comparison to optimizing either the transmission power or the frame length. The effect of different modulation and coding schemes on EPG is investigated. We consider that all the channel gains, γ_s, γ_p, γ_{ps}, and γ_{sp} follow an exponential distribution.

TABLE I
SIMULATION PARAMETERS.

Parameter Name	Value
Wireless channels	Rayleigh, Slow Flat Fading
P_p	20 dB
P_{pk}	20 dB
ϵ	0.6
Primary Rate (R_p)	0.5 symbol / sec
σ_{sp}^2, σ_{ps}^2	1
σ_s^2	1, 4, 5
Modulation	4 QAM & 16 QAM
Coding	UnCoded & Turbo

Figure 1 evaluates EPG versus ϵ under several optimization variables and different SU's channel's parameters ($\sigma_s^2 = 4$, $\sigma_s^2 = 5$). The notations in the figure are as follows, 'EPG' represents the scenario of optimizing EPG with respect to both P_s and L, 'EPGPs' represents the scenario where we optimize EPG with respect to only P_s, and 'EPGL' represents the scenario where we optimize EPG with respect to only L. It is clear that better channel quality (higher σ_s^2) results in an improved EPG. Increasing ϵ, hence increasing Q_I, results in improving EPG up to a certain value Q_I, where $P_{pk} < Q_I$. It is also observed that 'EPG' achieves better performance than 'EPGL' and 'EPGPs' for all values of ϵ. Finally, we observe that 'EPGL' increases after increasing ϵ, hence P^{st}, over a certain threshold. This means that fixing the power reduces the system's performance when optimizing L.

Figure 2 evaluates the EPG versus P_s, using optimal L, under several coding and modulation schemes, i.e., Uncoded, Turbo coded, 4QAM and 16 QAM, and several values of $P_t = 0.1, 0.3, 0.5$. It is noted that the Turbo coded communication achieves lower EPG in comparison to the uncoded

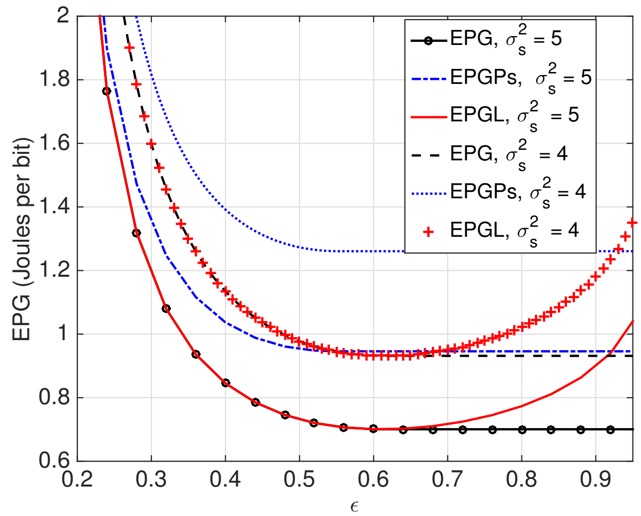

Fig. 1. $\mathcal{E}_e$ performance versus ϵ for $\sigma_s^2 = 4$ and $\sigma_s^2 = 5$.

communication. We also observe that higher modulation constellation (16 QAM) achieves worse EPG compared to the lower modulation constellation (4 QAM). However, at high value of the transmission power, all schemes approach similar EPG performance. Increasing P_t power increases the value of EPG. It is also noted that low values of P_t increases the tolerance of EPG toward the increment of P_s.

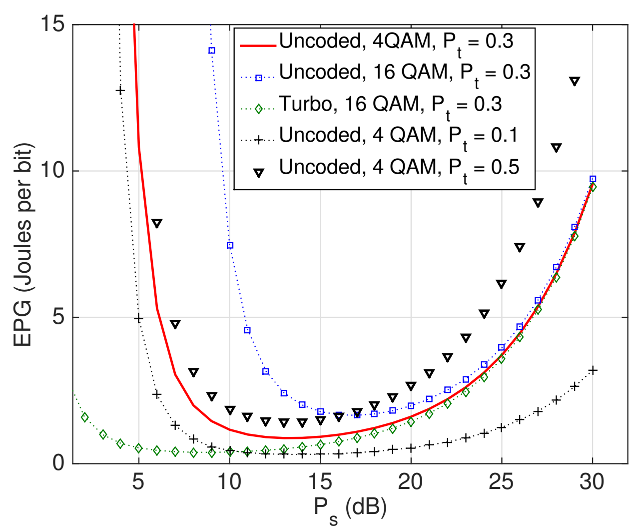

Fig. 2. $\mathcal{E}_e$ performance versus P_s and several modulation and coding schemes.

Figure 3 evaluates the EPG versus L, using optimal P_s, under several coding and modulation schemes, i.e., Uncoded, Turbo coded, 4QAM and 16 QAM, and several values of P_t. The change of the $\mathcal{E}_e$ structure is observed via changing the modulation scheme and power constraint P_{pk}. Inline with the results in Fig. 2, it is noted that the Turbo coded communication achieves lower EPG in comparison to the uncoded communication. We also observe that higher modulation constellation (16 QAM) achieves worse EPG compared to the lower modulation constellation (4 QAM). Increasing P_t power increases the value of EPG.

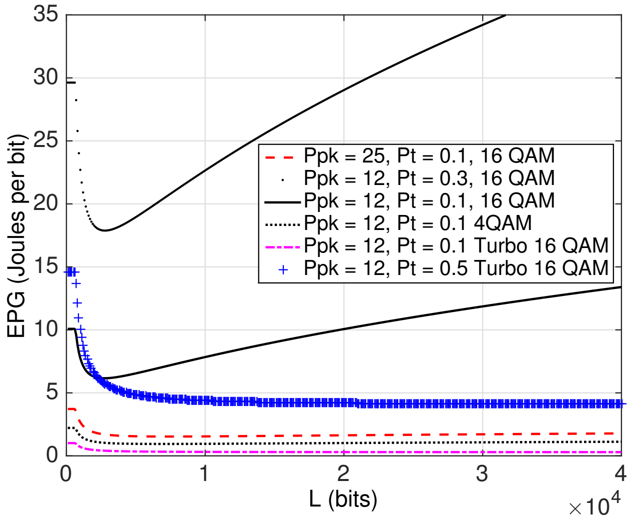

Fig. 3. $\mathcal{E}_e$ performance versus the frame length, L, for several modulation, coding schemes, P_t, and P_{pk}.

Figure 4 evaluates the EPG versus both P_s and L, under several MAC layer overhead, L_0, and channel quality, γ_s. This figure verifies the strict pseudo-convexity and strict quasi-convexity structures of the EPG with respect to each variable; however, it is not a convex structure. From Fig. 4 it is clear that EPG has a unique joint global minimum for all P_s and L which inline with the analytical result in the previous section. We also observe that changing L_0 has a different impact on the structure of EPG in comparison to changing γ_s. It is noted that the structure of EPG for $L_0 = 400$ and $L_0 = 1$ are similar to that in Fig. 2 and Fig. 3.

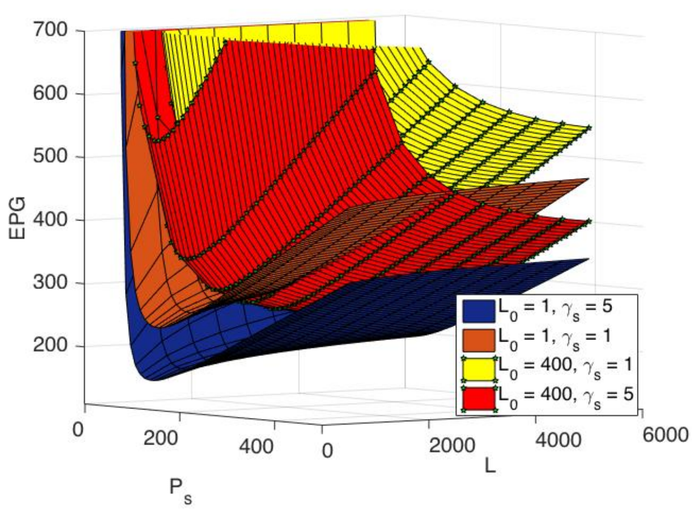

Fig. 4. $\mathcal{E}_e$ performance versus for all P_s and L given different channel quality, γ_s, and MAC overhead (L_0).

VI. Conclusion

In this work, we considered a spectrum-sharing model where we minimized the energy per good bit of the secondary user while protecting the primary user. A strict pseudo-convex structure of the problem is verified. Analytical expressions of the optimal power and frame length were provided. We also proposed an alternating algorithm to guarantee a global optimal solution. It is shown that optimizing the problem with respect to the power and the frame length, considerably, improves the system's performance in compared to conventional methods.

Appendix A

It is clear that $\nabla\mathcal{E}_e(P_s^*) = 0$ iff $\nabla\mathcal{E}_n(P_s^*)\mathcal{E}_d(P_s^*) - \mathcal{E}_n(P_s^*)\nabla\mathcal{E}_d(P_s^*) = 0$. Then, $\nabla\mathcal{E}_n(P_s^*) = \mathcal{E}_e(P_s^*)\nabla\mathcal{E}_d(P_s^*)$. Utilizing the properties of $\mathcal{E}_n$ function,

$$\begin{aligned}
\mathcal{E}_n(P_s) &= \mathcal{E}_n(P_s^*) + \nabla\mathcal{E}_n(P_s^*)(P_s - P_s^*) && \text{(24a)}\\
&= \mathcal{E}_n(P_s^*) + \mathcal{E}(P_s^*)\nabla\mathcal{E}_d(P_s^*)(P_s - P_s^*) && \text{(24b)}\\
&\geq \mathcal{E}_n(P_s^*) + \mathcal{E}(P_s^*)(\mathcal{E}_d(P_s) - \mathcal{E}_d(P_s^*)) && \text{(24c)}\\
&= \mathcal{E}_n(P_s^*) + \mathcal{E}(P_s^*)\mathcal{E}_d(P_s) - \mathcal{E}_n(P_s^*) && \text{(24d)}\\
\implies & \frac{\mathcal{E}_n(P_s)}{\mathcal{E}_d(P_s)} \geq \mathcal{E}(P_s^*), && \text{(24e)}
\end{aligned}$$

where (24a) follows from the convex property of $\mathcal{E}_n$ and (24c) is due to the concave structure of $\mathcal{E}_d(P_s)$.

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